

Exponential Functions and Sequences

Name _____ Class _____

1 If $f(1) = 8$ and $f(n) = \frac{1}{2}f(n-1) + 2$ then which of the following is the value of $f(3)$?

- (1) 5 (3) 3
 (2) 7 (4) 6

$f(1) = 8, f(2) = 6, f(3) = 5$
 $f(2) = \frac{1}{2} \cdot f(1) + 2 = \frac{1}{2}(8) + 2 = 4 + 2 = 6$
 $f(3) = \frac{1}{2} \cdot f(2) + 2 = \frac{1}{2}(6) + 2 = 3 + 2 = 5$

2 The first two terms of an arithmetic sequence are 6 and 11, respectively. Which of the following represents the 14th term of the sequence?

- (1) 71 (3) 76
 (2) 66 (4) 61

6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71
 $+5$
 OR:
 $f(n) = 5n + 1 \rightarrow f(14) = 5(14) + 1 = 71$

3 In order to convert 3.6 meters into yards, Evin used the following product:

$$3.6 \text{ meters} \times \frac{100 \text{ centimeters}}{1 \text{ meter}} \times \frac{2.54 \text{ centimeters}}{1 \text{ inch}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ yard}}{3 \text{ feet}}$$

Which of the following ratios was written incorrectly in this conversion process?

- (1) $\frac{100 \text{ centimeters}}{1 \text{ meter}}$ (3) $\frac{1 \text{ foot}}{12 \text{ inches}}$
 (2) $\frac{2.54 \text{ centimeters}}{1 \text{ inch}}$ (4) $\frac{1 \text{ yard}}{3 \text{ feet}}$

Discussed in class

4 Given the arithmetic sequence shown below, write a recursive rule and an explicit rule for the sequence.

-4, 4, 12, 20, ...

Recursive Rule

Explicit Rule

$$f(n+1) = f(n) + 8$$

for $f(1) = -4$

$$a_n = a_1 + (n-1)d$$

$$a_n = -4 + (n-1)8$$

$$a_n = -4 + 8n - 8$$

$$f(n) = 8n - 12$$

5 A sequence is defined by the recursive formula $a_1 = -6$ and $a_n = a_{n-1} + 2n$. Write out the next three terms of this sequence. Show the work that leads to your terms.

$a_1 = -6$
 $a_n = a_{n-1} + 2n$
 $a_2 = a_1 + 2(2)$
 $a_2 = -6 + 4 = -2$

$a_2 = -2$

$a_3 = a_2 + 2(3)$
 $a_3 = -2 + 6 = 4$

$a_3 = 4$

$a_4 = a_3 + 2(4)$
 $a_4 = 4 + 8$

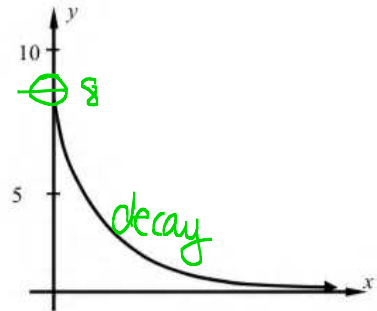
$a_4 = 12$

6 Which of the following is the equation of an increasing exponential function?

- (1) ~~$y = 4(0.75)^x$~~ $b < 1$ (3) $y = 5x - 2$ linear
 (2) $y = 7\left(\frac{3}{2}\right)^x$ $b > 1$ (4) $y = 4x^2$ quadratic

7 Which of the following could be the equation of the exponential function shown?

- (1) ~~$y = 3(0.9)^x$~~ decay, $b < 1 \rightarrow y\text{-int} = 3$
 (2) ~~$y = 7(1.2)^x$~~ growth, $b > 1$
 (3) $y = 8(0.5)^x$ decay, $b < 1 \rightarrow y\text{-int} = 8$
 (4) ~~$y = 4(2.5)^x$~~ growth, $b > 1$



8 People are entering a stadium at a steady rate of 32 people per minute. When the gates open, there are already 46 people in the stadium. No one leaves the stadium for the first hour after the gates have opened.

a) How many people will be in the stadium 30 minutes after it opens? Show the calculations that lead to your answer.

$$32(30) + 46 = \boxed{1006 \text{ people}}$$

b) Write a linear equation for the number of people, n , as a function of the time in minutes, m , since the gates were opened.

$$\boxed{n(m) = 32m + 46}$$

c) After one hour, no additional people enter, but some start to leave. If it takes them a total of 4 hours to leave the stadium, what is the average rate at which they leave, in people per hour? Show the calculations that lead to your answer.

$$\begin{aligned} n(60) &= 32(60) + 46 \\ n(60) &= 1920 + 46 \\ n(60) &= 1966 \end{aligned} \quad \frac{1966}{4} = \boxed{491.5 \text{ people per hour}}$$

9 The population of Ketcham High School has been decreasing by 5% each year. If its population is currently 2,600 students, which of the following is closest to its population two years from now?

- (1) 2,340 (3) 2,470
 (2) 2,347 (4) 2,590

$$A(t) = P(1+r)^t \quad A(t) = 2600(1-0.05)^t$$

$$A(2) = 2600(0.95)^2 \approx 2347$$

10 The population of bacteria is increasing at a rate of 7.5% per hour. If there were originally 275 bacteria, which of the following equations models the population of bacteria h hours after the original 275 bacteria were measured?

(1) $P = 275(7.5)^h$ (3) $P = 1.075h + 275$

(2) $P = 7.5(275)^h$ (4) $P = 275(1.075)^h$

$P(h) = A(1+r)^h$
 $P(h) = 275(1+0.075)^h$

$P(h) = 275(1.075)^h$

11 A radioactive material has a mass given by $m(t) = 126(0.84)^t$, where mass is in grams and the time, t , is in years. Which of the following gives the average rate of change of the mass over the interval $2 \leq t \leq 5$ years?

(1) -12.1 grams per year (3) 18.3 grams per year

(2) -36.2 grams per year (4) 28.9 grams per year

$m(2) = 126(0.84)^2 = 88.9056$

$m(5) = 126(0.84)^5 = 52.69470474$

$\frac{m(5) - m(2)}{5 - 2} = \frac{-36.210895256}{3} \approx -12.1$

12 An exponential function, f , is shown in the table below. Determine an equation for it in

$f(x) = a \cdot b^x$ form.

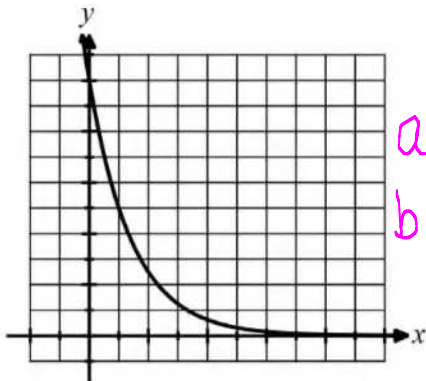
$a = 13$ (initial value)

$b = 3$ ($\frac{39}{13} = 3, \frac{117}{39} = 3$)

$f(x) = 13 \cdot (3)^x$

x	0	1	2	3
$f(x)$	13	39	117	351

13 Determine the equation of the exponential function shown graphed below.



$a = 10$ (initial value)

$b = \frac{1}{2}$

($\frac{5}{10} = \frac{1}{2}, \frac{2.5}{5} = \frac{1}{2}$)

$f(x) = 10(\frac{1}{2})^x$

14 A geometric sequence is defined using the following rule:

$f(1) = 4$ and $f(n) = f(n-1) \cdot -2$

Find the sixth term of the sequence. Show how you arrived at your answer.

$f(1) = 4, f(n) = f(n-1) \cdot -2$

$f(2) = f(1) \cdot -2 = 4 \cdot -2 = -8$

$f(3) = f(2) \cdot -2 = -8 \cdot -2 = 16$

$f(4) = f(3) \cdot -2 = 16 \cdot -2 = -32$

$f(5) = f(4) \cdot -2 = -32 \cdot -2 = 64$

$f(6) = f(5) \cdot -2 = 64 \cdot -2 = -128$

-128

15 Which is larger, the 10th term of an arithmetic sequence that starts with the terms 0 and 100 or the 10th term of a geometric sequence that starts with the terms 5 and 10? Show your work that justifies your answer.

A: 0, 100, 200, 300, 400, 500, 600, 700, 800, 900

G: 5, 10, 20, 40, 80, 160, 320, 640, 1280, 2560

10th Term of Geometric Sequence

16 The first three terms of a geometric sequence are shown below. Write a recursive rule and explicit rule for this sequence.

216, 144, 96

Recursive Rule

$$f(n) = f(n-1) \cdot \frac{2}{3}$$

for $f(1) = 216$

$$r = \frac{144}{216} = \frac{2}{3}$$

$$r = \frac{96}{144} = \frac{2}{3}$$

Explicit Rule

$$f(n) = f(1) \cdot r^{n-1}$$

$$f(n) = 216 \cdot \left(\frac{2}{3}\right)^{n-1}$$

17 If $f(1) = 4$ and $f(n+1) = -3f(n) + 2$, then find $f(4)$.

~~$f(1) = 4$~~ $f(n+1) = -3f(n) + 2$

$f(2) = -3 \cdot f(1) + 2 = -3(4) + 2 = -12 + 2 = -10$

$f(3) = -3 \cdot f(2) + 2 = -3(-10) + 2 = 30 + 2 = 32$

$f(4) = -3 \cdot f(3) + 2 = -3(32) + 2 = -96 + 2 = -94$

18 A function passes through the points (0,8) and (1,24).

a) Write the equation of a linear function that passes through these two points and write the equation of an exponential function that passes through these two points. Do not use a calculator for it.

Linear Equation

(0,8), (1,24)

$$m = \frac{24-8}{1-0} = \frac{16}{1} = 16$$

$$y = 16x + b$$

$b = 8$ b/c (0,8) is the y-int.

$$y = 16x + 8$$

Exponential Equation

$$y = a \cdot b^x$$

$a \rightarrow$ initial value is 8

$b \rightarrow$ growth rate
 $\left(\frac{y_2}{y_1} = \frac{24}{8} = 3\right)$

$$y = 8(3)^x$$

b) How much greater is the exponential function's value at $x = 5$ than the linear function's value? Show how you arrived at your answer.

$$y = 16(5) + 8$$

$$y = 80 + 8$$

$$y = 88$$

$$y = 8 \cdot (3)^5$$

$$y = 8(243)$$

$$y = 1944$$

$$1944 - 88 = 1856$$