Polynomials and Quadratic Functions Review

Name ______________________________ Class __________

Factor the following expressions:

1 $y^2 - y - 30$

$(y-6)(y+5)$

2 $x^4 - 1$

$(x^2-1)(x^2+1)$

$\boxed{(x-1)(x+1)(x^2+1)}$

3 $3x^2 + 11x + 6$

$\boxed{(3x+2)(x+3)}$

$2x$

$9x$

4 $5x^2 + 60x + 100$

$5(x^2+12x+20)$

$\boxed{5(x+10)(x+2)}$

5 $2x^5y - 32xy$

$2xy(x^4-16)$

$2xy(x^2+4)(x^2-4)$

$\boxed{2xy(x^2+4)(x-2)(x+2)}$

6 $4x^4 - 29x^2 + 25$

$(4x^2-25)(x^2-1)$

$-25x^2$

$-4x^2$

$\boxed{(2x-5)(2x+5)(x-1)(x+1)}$

Solve the following equations:

7 $r^2 + 2r - 3 = 4$

$r^2+2r-7=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{(2)^2-4(1)(-7)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4+28}}{2}$

$x = \dfrac{-2 \pm \sqrt{32}}{2} = \dfrac{-2 \pm \sqrt{16}\sqrt{2}}{2} = \dfrac{-2 \pm 4\sqrt{2}}{2}$

$= -1 \pm 2\sqrt{2}$

$\boxed{\{-1+2\sqrt{2}, -1-2\sqrt{2}\}}$

8 $r^2 + 20r + 73 = -9$

$r^2+20r = -82$

$\left(\frac{20}{2}\right)^2 = (10)^2 = 100$

$r^2+20r+100 = -82+100$

$(r+10)^2 = 18$

$r+10 = \pm\sqrt{18}$

$r+10 = \pm\sqrt{9}\sqrt{2}$

$r+10 = \pm 3\sqrt{2}$

$r = -10 \pm 3\sqrt{2}$

$\boxed{\{-10+3\sqrt{2}, -10-3\sqrt{2}\}}$

9 For which function defined by a polynomial are the zeros of the polynomial -4 and -6?

(1) $y = x^2 - 10x - 24$

(2) $y = x^2 + 10x + 24$

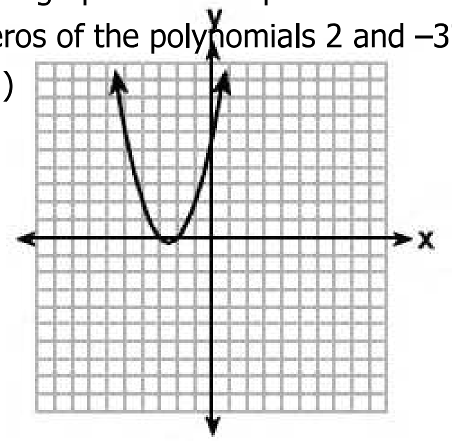
(3) $y = x^2 + 10x - 24$

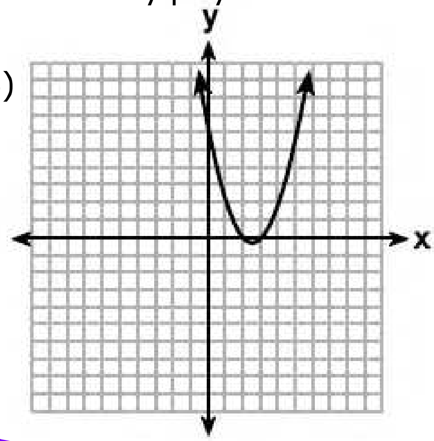
(4) $y = x^2 - 10x + 24$

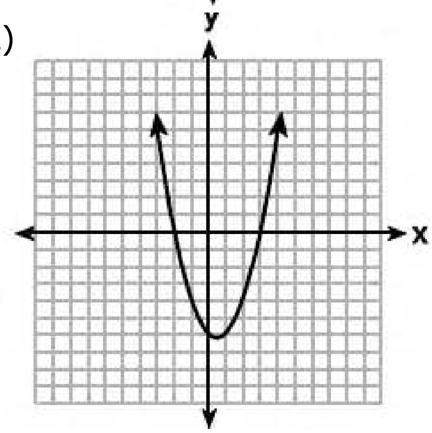
$y = (x+4)(x+6)$

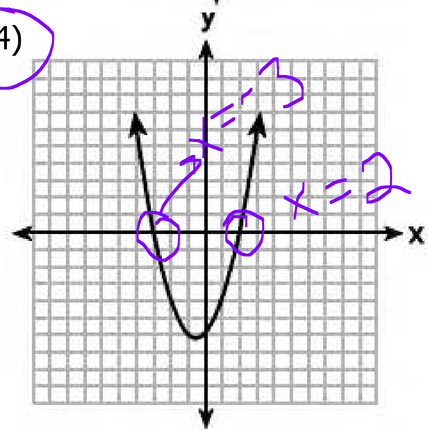
$y = x^2+10x+24$

10 The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and −3?

(1)


(3)


(2)


(4)


11 Which equation has roots of -3 and 5?

(1) $x^2+2x-15=0$ (3) $x^2+2x+15=0$

(2) $x^2-2x-15=0$ (4) $x^2-2x+15=0$

$(x+3)(x-5)=0$

$x^2-5x+3x-15=$

12 The function $f(x)$ is given below.

$f(x)=x^2+2x-3$

a) Describe the effect on the graph of $f(x)$, if $g(x)=f(x-5)$.

It gets shifted to the right 5 units

b) Show that the vertices of $f(x)$ and $g(x)$ support your description.

Put f(x) in vertex form:

$f(x)=x^2+2x+1-3-1$

$f(x)=(x+1)^2-4$

$\boxed{\text{vertex: } (-1,-4)}$

$g(x)=f(x-5)$

$g(x)=(x-5)^2+2(x-5)-3$

$g(x)=x^2-10x+25+2x-10-3$

$g(x)=x^2-8x+12$

Now, put g(x) in vertex form:

$g(x)=x^2-8x+16+12-16$

$g(x)=(x-4)^2-4$

vertex: (4,-4)

Since the vertex of f(x) is (-1,-4) and the vertex of g(x) is (4,-4), the vertex shifted 5 units right.

13 A model rocket is launched from a platform in a flat, level field and lands in the same field. The height of the rocket follows the function, $f(x) = -16x^2 + 150x + 5$, where f(*x*) is the height, in feet, of the rocket and *x* is the time, in seconds, since the rocket is launched.

a) Determine the maximum height, to the *nearest tenth of a foot*, the rocket reaches.

$f(x) = -16x^2 + 150x + 5$

$x = \frac{-b}{2a} = \frac{-150}{2(-16)} = \frac{-150}{-32} = 4.6875$

$f(4.6875) = \boxed{356.6\text{ft}}$

b) Determine the length of time, to the *nearest tenth of a second*, from when the rocket is launched until it hits the ground.

$0 = -16x^2 + 150x + 5$

$x = \frac{-150 \pm \sqrt{(150)^2 - 4(-16)(5)}}{2(-16)}$

$x = \frac{-150 \pm \sqrt{22820}}{-32}$

$x \approx \cancel{-0.03}$

$\boxed{x \approx 9.4\text{s}}$

14 Consider the equation $x^2 - 2x - 6 = y + 2x + 15$ and the function $f(x) = 4x^2 - 16x - 84$ in the following questions.

a) Show that the graph of the equation $x^2 - 2x - 6 = y + 2x + 15$ has *x*-intercepts at $x = -3$ and 7.

$x^2 - 2x - 6 = y + 2x + 15$

$\quad -2x - 15 \quad\quad -2x - 15$

$x^2 - 4x - 21 = y$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$\boxed{\{7, -3\}}$

b) Show that the zeroes of the function $f(x) = 4x^2 - 16x - 84$ are the same as the *x* values of the *x*-intercepts for the graph of the equation in part (a).

$f(x) = 4x^2 - 16x - 84$

$0 = 4x^2 - 16x - 84$

$0 = 4(x^2 - 4x - 21)$

$0 = 4(x-7)(x+3)$

$\boxed{\{7, -3\}}$

c) Explain how this function is different from the equation in part (a).

$f(x) = 4(x^2 - 4x - 21)$

$y = x^2 - 4x - 21$

f(x) is y after a vertical stretch of scale factor 4.

d) Identify the vertex of the graphs of each by rewriting the equation and function in the completed-square form, $f(x) = a(x-h)^2 + k$. Show your work. What is the same about the two vertices? How are they different? Explain why there is a difference.

$y = x^2 - 4x - 21$

$y = x^2 - 4x + 4 - 21 - 4$

$y = (x-2)^2 - 25$

vertex: $(2, -25)$

$f(x) = 4x^2 - 16x - 84$

$f(x) = 4(x^2 - 4x + 4) - 84 - 16$

$f(x) = 4(x-2)^2 - 100$

$(2, -100)$

The vertex has the same x-value, but the y-value is 4 times larger, due to the vertical stretch s.f. 4

15 An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t, and the height of the arrow in meters, h, is given by:

$$h(t) = -4.9t^2 + 29.4t + 2.5$$

a) Complete the square for this function.

$h(t) = -4.9(t^2 - 6t + 9) + 2.5 + 44.1$

$h(t) = -4.9(t-3)^2 + 46.6$

b) What is the maximum height of the arrow? Explain how you know.

46.6m - The vertex occurs at (3, 46.6)

c) How long does it take the arrow to reach its maximum height? Explain how you know.

3 sec.

The vertex occurs at (3, 46.6)

d) What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds? Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds and explain the difference in the context of the problem.

$h(1) = 27$

$h(2) = 41.7$

$\frac{41.7-27}{2-1}$

$\frac{14.7}{1} = \boxed{14.7 \text{ m/s}}$

$h(3) = 46.6$

$h(2) = 41.7$

$\frac{46.6-41.7}{3-2}$

$\frac{4.9}{1} = \boxed{4.9 \text{ m/s}}$

The avg rate of change from 1 sec to 2 sec is faster than from 2 to 3 sec. because gravity is slowing the arrow down as it reaches its vertex.

e) How long does it take the arrow to hit the ground? Show your work.

$h(t) = -4.9(t-3)^2 + 46.6$

$0 = -4.9(t-3)^2 + 46.6$

$-46.6 = -4.9(t-3)^2$

$\frac{46.6}{4.9} = (t-3)^2$

$\pm\sqrt{\frac{46.6}{4.9}} = t-3$

$t = 3 \pm \sqrt{\frac{46.6}{4.9}}$

$t = 6.085$ or ~~$t = -0.085$~~

$\boxed{6.085}$

f) What does the constant term in the original equation tell you about the arrow's flight?

It started at a height of 2.5m.