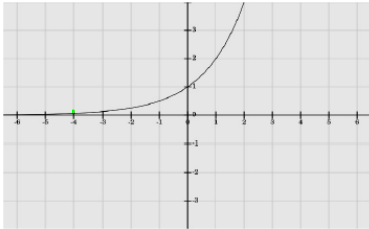# General Function Review

Name _____ Class _____

1 Compare the following three functions:

    i) A function f is represented by the graph below.

    ii) A function g is represented by the following equation:

$$g(x) = (x-6)^2 - 36$$

    iii) A linear function h is represented by the following table

x	−1	0	1	2	3	4	5	6	7
$h(x)$	10	12	14	16	18	20	22	24	26

For each of the following, evaluate the three expressions given and identify which expression has the largest value and which has the smallest value. Show your work.

a) $f(0)$, $g(0)$, $h(0)$

$f(0) = 1$

$g(0) = (0-6)^2 - 36$
$g(0) = 36 - 36$
$g(0) = 0$
↑ smallest value

$h(0) = 12$
↑ Largest Value

b) $\dfrac{f(4) - f(2)}{4 - 2}$ $\dfrac{g(4) - g(2)}{4 - 2}$ $\dfrac{h(4) - h(2)}{4 - 2}$

$f(x) = 2^x$
$f(4) = 2^4 = 16$
$f(2) = 2^2 = 4$

$\dfrac{16 - 4}{4 - 2}$

$\dfrac{12}{2} = 6$ ← largest

$g(4) = (4-6)^2 - 36$
$g(4) = (-2)^2 - 36 = -32$

$g(2) = (2-6)^2 - 36$
$g(2) = (-4)^2 - 36 = -20$

$\dfrac{-32 - -20}{4 - 2} = \dfrac{-12}{2} = -6$ ← smallest

$h(4) = 20$
$h(2) = 16$

$\dfrac{20 - 16}{4 - 2} = \dfrac{4}{2} = 2$

2 Solve the following equation by factoring.

$8x^2 + 21 = -59x$

$\underline{+59x \qquad +59x}$

$8x^2 + 59x + 21 = 0$

$(8x + 3)(x + 7) = 0$

$8x + 3 = 0 \quad | \quad x + 7 = 0$
$8x = -3 \quad | \quad x = -7$
$x = -3/8$

$\left\{ -\dfrac{3}{8}, -7 \right\}$

3 Label each graph with the function it represents; choose from those listed below. Do not use a calculator.
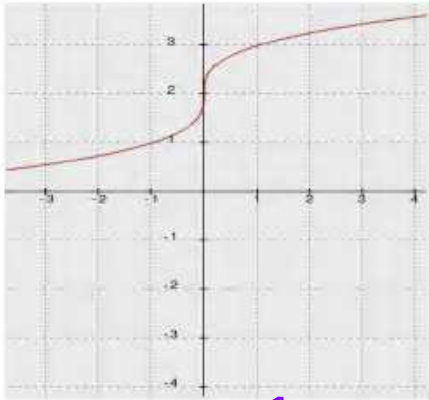
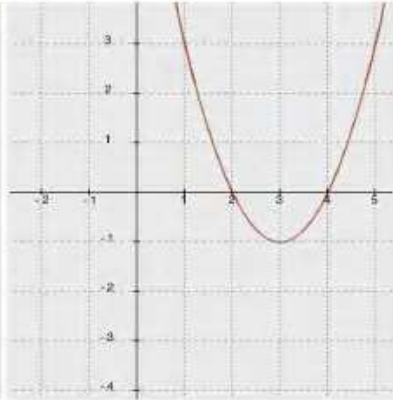$$f(x) = 3\sqrt{x} \qquad g(x) = \frac{1}{2}\sqrt[3]{x} \qquad h(x) = -5x^2$$

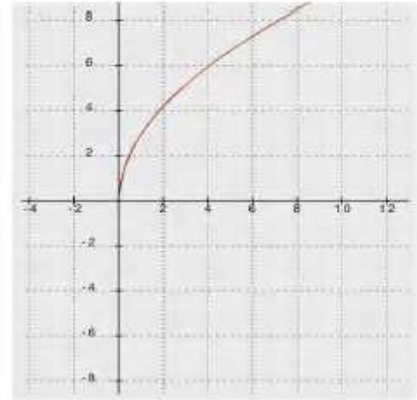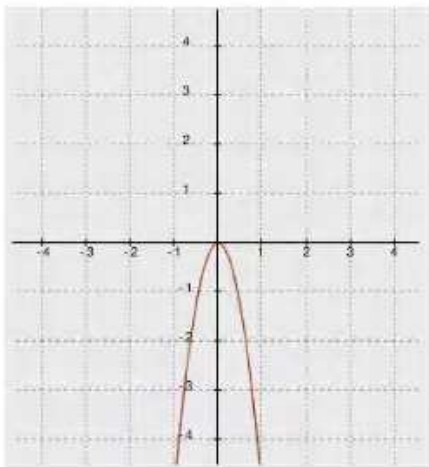$$k(x) = \sqrt{x+2} - 1 \qquad m(x) = \sqrt[3]{x} + 2 \qquad n(x) = (x-3)^2 - 1$$
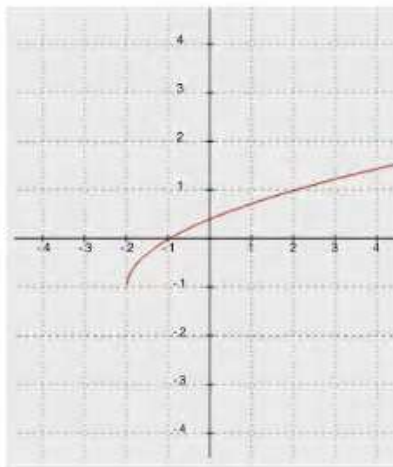
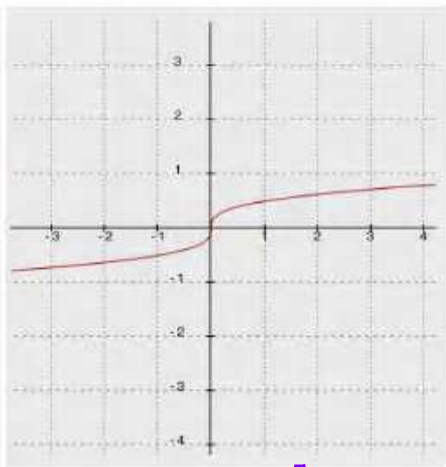Function $m(x) = \sqrt[3]{x} + 2$

Function $n(x) = (x-3)^2 - 1$

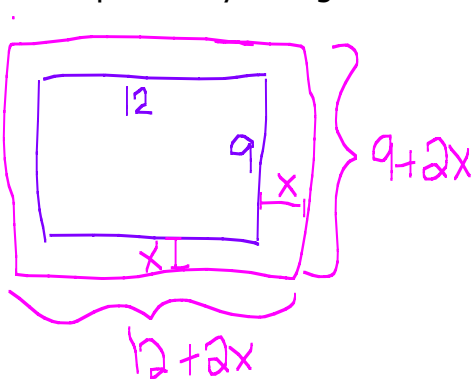Function $f(x) = 3\sqrt{x}$

Function $h(x) = -5x^2$

Function $k(x) = \sqrt{x+2} - 1$

Function $g(x) = \frac{1}{2}\sqrt[3]{x}$

4 A concrete border of uniform width is constructed around a rectangular flower bed that is 12 ft. by 9 ft. Find the width of the border if the area formed by the flower bed and the border is 154 sq. ft. Only an algebraic solution will be accepted.

$$A = L \cdot W$$
$$154 = (12+2x)(9+2x)$$
$$154 = 108 + 24x + 18x + 4x^2$$
$$154 = 4x^2 + 42x + 108$$
$$0 = 4x^2 + 42x - 46$$
$$0 = 2(2x^2 + 21x - 23)$$
$$0 = 2(2x + 23)(x - 1)$$

$$x = -\frac{23}{2} \text{ omit} \qquad x = 1$$

$1 ft$

12

9

x

9+2x

12+2x

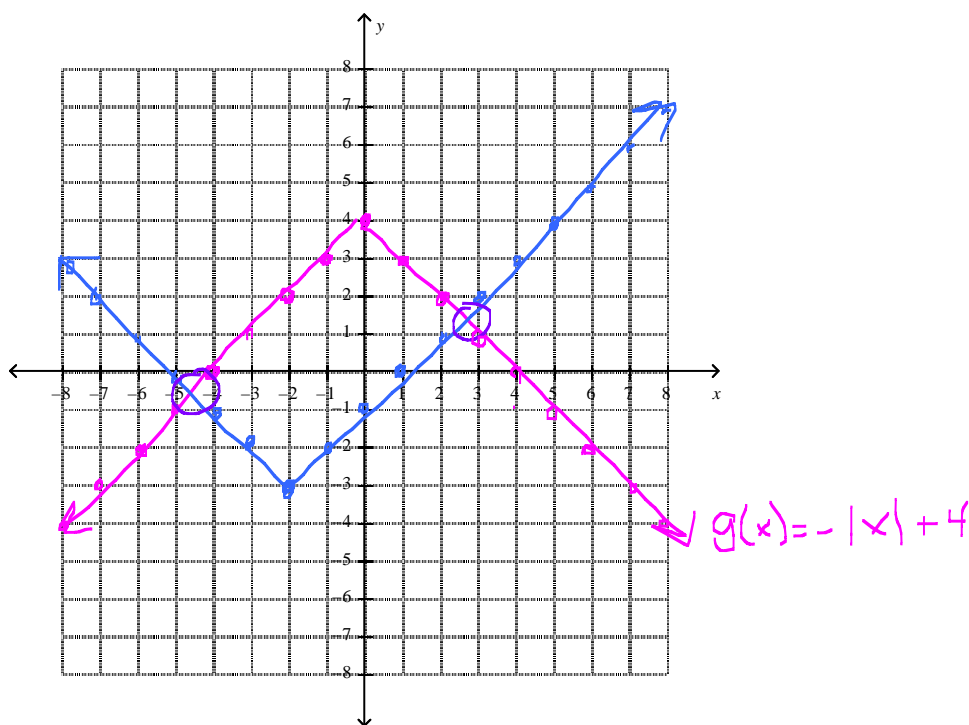**5** Given $h(x) = |x+2| - 3$ and $g(x) = -|x| + 4$.

a) Describe how to obtain the graph of g from the graph of $p(x) = |x|$ using transformations.

- Reflect over x·axis
- shift up 4

b) Describe how to obtain the graph of h from the graph of $p(x) = |x|$ using transformations.

- left 2
- down 3

c) Sketch the graphs of $h(x)$ and $g(x)$ on the same coordinate plane.

$g(x) = -|x| + 4$

d) Use your graphs to estimate the solutions to the equation:

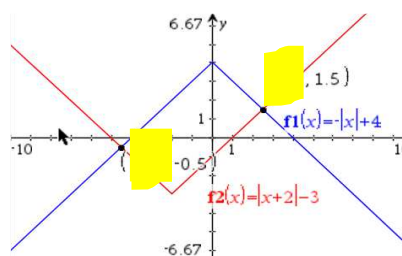$|x+2| - 3 = -|x| + 4$

Explain how you got your answer.

$x = 2.5, \quad x = -4.5 \quad \{2.5, -4.5\}$

Its where the graphs intersect

e) Were your estimations you made in part (d) correct? If yes, explain how you know. If not explain why not.

Yes - I used my calculator to check.

6 A boy bought 6 guppies at the beginning of the month. One month later the number of guppies in his tank had doubled. His guppy population continued to grow in this same manner. His sister bought some tetras at the same time. The table below shows the number of tetras, t, after n months have passed since they bought the fish.

n, months	0	1	2	3
t, tetras	8	16	24	32

a) Create a function g to model the growth of the boy's guppy population, where $g(n)$ is the number of guppies at the beginning of each month, and n is the number of months that have passed since he bought the 6 guppies. What is a reasonable domain for g in this situation?

$$g(n) = a \cdot b^n \qquad a = 6 \text{ (initial value)}, \quad b = 2 \text{ (doubles each month)}$$

$$\boxed{g(n) = 6 \cdot 2^n}$$

b) How many guppies will there be one year after he bought the 6 guppies?

$$g(12) = 6 \cdot 2^{12} = \boxed{24,576 \text{ guppies}}$$

c) Create an equation that could be solved to determine how many months after he bought the guppies there will be 100 guppies.

$$100 = 6(2)^n$$

d) Use graphs or tables to approximate a solution to the equation from part (c). Explain how you arrived at your estimate.

n	0	1	2	3	4	5
$g(n)$	6	12	24	48	96	192

Between 4 and 5 months

e) Create a function, t, to model the growth of the sister's tetra population, where $t(n)$ is the number of tetras after n months have passed since she bought the tetras.
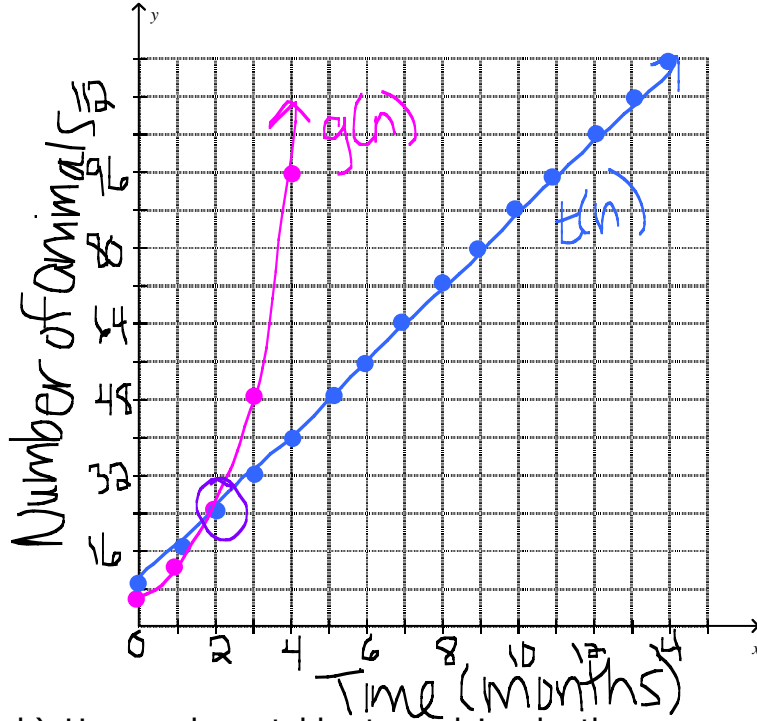
$$m = 8$$
$$b = 8 \qquad t(n) = 8n + 8$$

f) Compare the growth of the sister's tetra population to the growth of the guppy population. Include a comparison of the average rate of change for the functions that model each population's growth over time.

The guppy population increases at an exponential rate, so the average rate of change isn't constant.

The tetra population increases at a constant rate of change of 8 tetras per month.

g) Use graphs to estimate the number of months that will have passed when the population of guppies and tetras will be the same.

Number of animals (y-axis): 16, 32, 48, 64, 80, 96, 112

Time (months) (x-axis): 0, 2, 4, 6, 8, 10, 12, 14

g(n), t(n)

After 2 months

h) Use graphs or tables to explain why the guppy population will eventually exceed the tetra population even though there were more tetras to start with.

An exponential growth rate is much more drastic than a linear growth rate.

7 The graph of a piecewise function f is shown to the right. The domain of f is $-3 \le x \le 3$.

a) Create an algebraic representation for f. Assume that the graph of f is composed of straight line segments.

$$f(x) = \begin{cases} x, & -3 \le x < -1 \\ -1, & -1 \le x < 0 \\ x-1, & 0 \le x < 2 \\ -x+3, & 2 \le x \le 3 \end{cases}$$
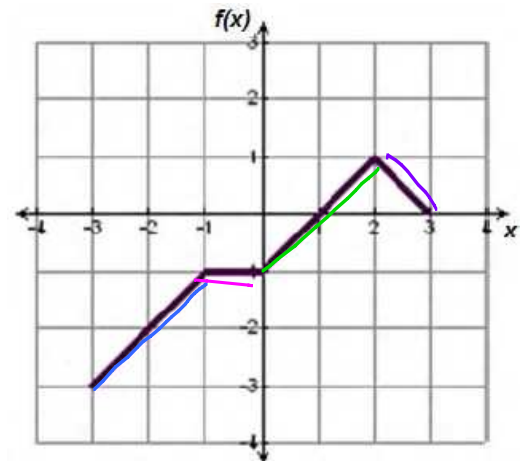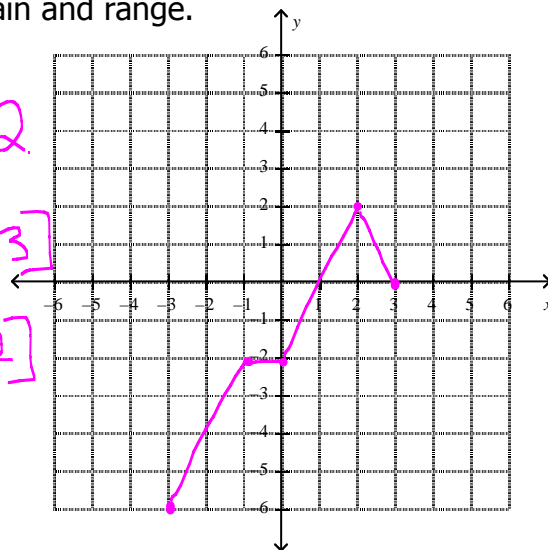
b) Sketch the graph of $y = 2f(x)$ and state the domain and range.

* Multiply each y-value by 2.

Domain: $-3 \leq x \leq 3$ or $[-3,3]$

Range: $-6 \leq y \leq 2$ or $[-6,2]$

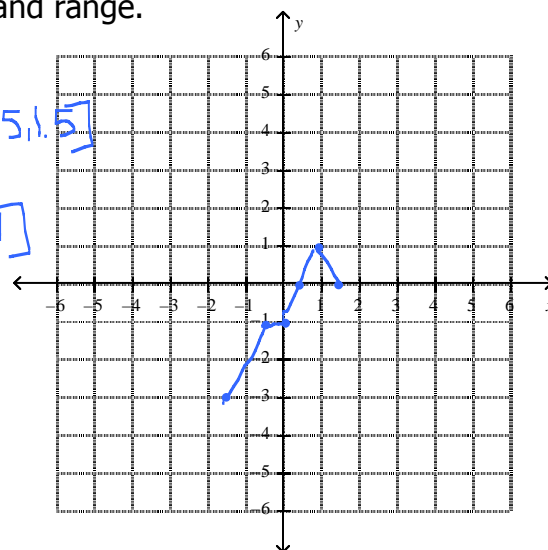c) How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

Range of $f(x)$: $[-3, 1]$

You must multiply the y values by k.
(will stretch with a scale factor of k.)

d) Sketch the graph of $y = f(2x)$ and state the domain and range.

Domain: $-1.5 \leq x \leq 1.5$ or $[-1.5, 1.5]$

Range: $-3 \leq y \leq 1$ or $[-3, 1]$

e) How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?

You must multiply each x-value
by $\frac{1}{k}$.
(will compress with a scale factor of $\frac{1}{k}$).

8 The tables below represent values for two functions, f and g, one absolute value and one quadratic.

a) Label each function as either absolute value or quadratic. Then explain mathematically how you identified each type of function.
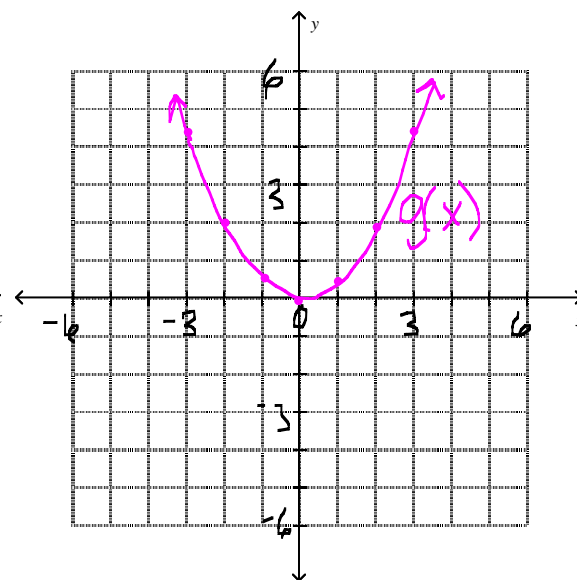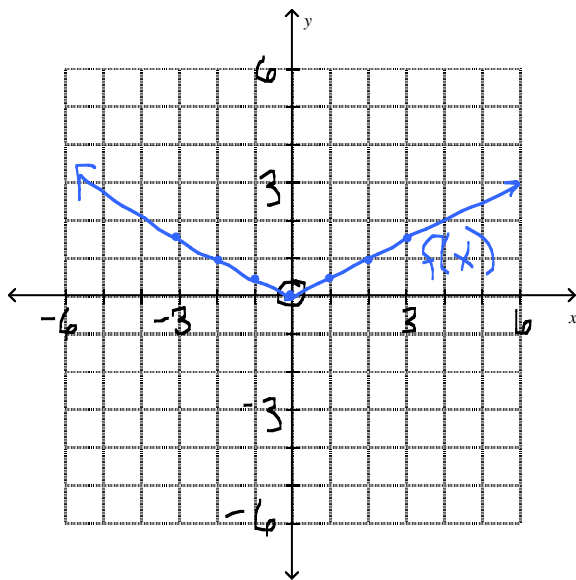
$f(x)$: _Absolute value_

x	$f(x)$
-3	1.5
-2	1
-1	0.5
0	0
1	0.5
2	1
3	1.5

$g(x)$: _Quadratic_

x	$g(x)$
-3	4.5
-2	2
-1	0.5
0	0
1	0.5
2	2
3	4.5

b) Represent each function graphically.

c) Represent each function algebraically.

Absolute value

$p(x) = |x|$

★ no shift

★ vertical compression

$\left.\begin{array}{l} f(x) = a|x| \\ (1, 0.5) \\ 0.5 = a|1| \\ 0.5 = 1a \\ a = 0.5 \end{array}\right.$

$f(x) = 0.5|x|$

Quadratic

$p(x) = x^2$

★ vertical compression

$g(x) = ax^2$

$(2, 2) \rightarrow 2 = a(2)^2$

$2 = 4a$

$a = 0.5$

$g(x) = 0.5x^2$