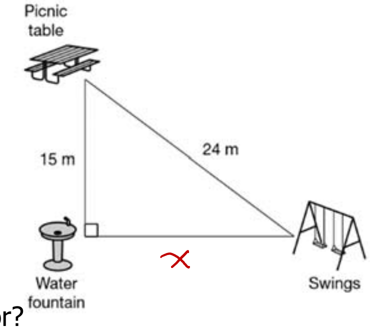
Unit 6 Review - Measurement Relationships

Name: ______________________________ Class: ______________

1.) The positions of the water fountain, the picnic table and the swings at a local park are shown below. Find the distance between the water fountain and the swings to the nearest tenth of a meter.



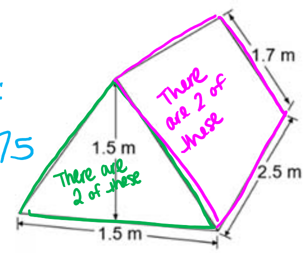
$15^2 + x^2 = 24^2$

$225 + x^2 = 576$

$x^2 = 351$

$\boxed{x \approx 18.7m}$

2.) Examine the tent. What is the surface area of the tent, including the ends and the floor?



2 Δ's

$A = \frac{1}{2}bh \cdot 2$

$A = b \cdot h$

$A = (1.5)(1.5)$

$A = 2.25m^2$

2 side ▭'s:

$A = b \cdot h \cdot 2$

$A = (2.5)(1.7)(2)$

$A = 8.5m^2$

Bottom Rectangle:

$A = b \cdot h$

$A = (1.5)(2.5)$

$A = 3.75m^2$

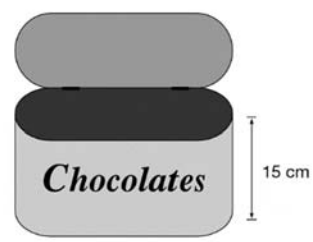
Total area:

$= 2.25 + 8.5 + 3.75$

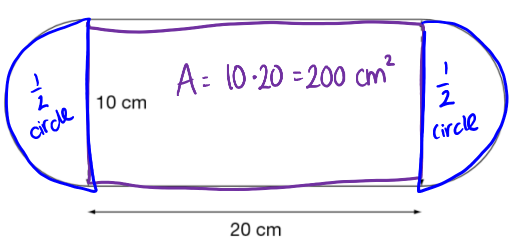
$= \boxed{14.5m^2}$

3.) Sweet Shapes is a company that makes chocolate. Each year, the company produces a new can for its specialty chocolates. This year's can is illustrated to the right. The top of the can swings open for easy access.

Derek makes a sketch of the bottom of the can and records the measurements below.



a.) Determine the area of the bottom of the can. Round to the nearest hundredth.



2 half circles = 1 full circle

$A = \pi r^2 \quad (r = 5)$

$A = \pi(5)^2$

$A = 25\pi$

Total area $= 200 + 25\pi$

$\approx \boxed{278.54 cm^2}$

b.) The can contains individually wrapped chocolates that each take up about 28 cm^3 of space. Determine how many chocolates a container of height 15 cm will hold.

① Find volume of can:

$V = A_{base} \cdot height$

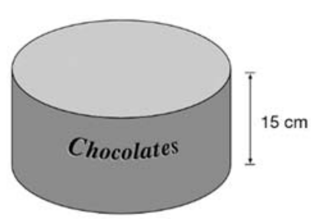
$= (278.54)(15)$

$= 4178.1 cm^3$

② Divide total volume by volume of one candy to find how many fit:

$\frac{4178.1}{28} = \boxed{149 \text{ candies}}$

c.) Next year, Sweet Shapes will produce a **cylindrical** can for the chocolates. The can will be filled completely with 75 wrapped chocolates, each with a volume of 19 cm^3. This can will also have a height of 15 cm. Determine the radius of this can, to the nearest tenth.



① Find total volume:

$75 \cdot 19 = 1425 cm^3$

② Use volume of a cylinder to find radius:

$V = \pi r^2 h$

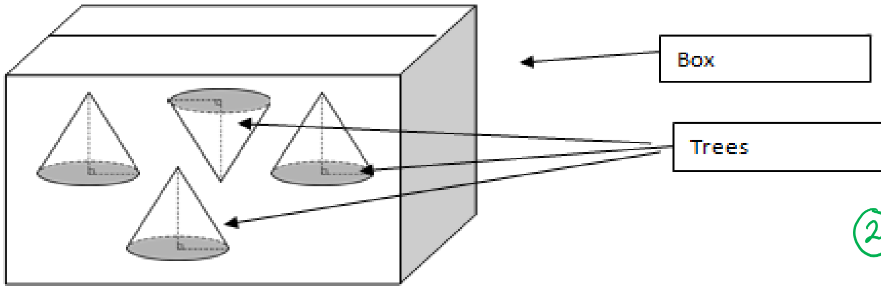
$\frac{V}{\pi h} = r^2$

$\sqrt{\frac{V}{\pi h}} = r$

$\sqrt{\frac{1425}{\pi \cdot 15}} = r$

$r = \sqrt{95/\pi} \approx \boxed{5.5 cm}$

4.) A company that makes glass trees for the holiday is shipping them in a box with 4 as shown below



① Find volume of box:

$V = l \cdot w \cdot h$

$V = (40)(20)(30)$

$V = 24000$

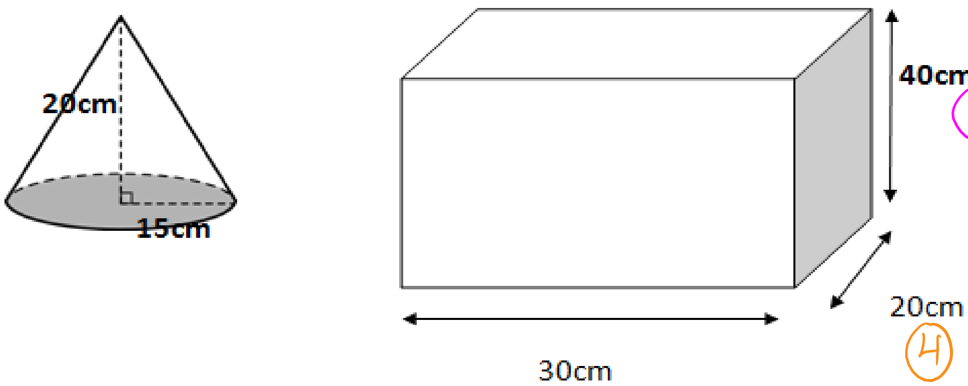
② Find volume of each tree:

$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi (15)^2 (20)}{3}$

$V = \frac{4500\pi}{3} = 1500\pi \text{ cm}^3$

The dimensions of the box and the trees are given below. If the company wants to fill the remaining space in the box with packing material what volume of packing material will they need ?



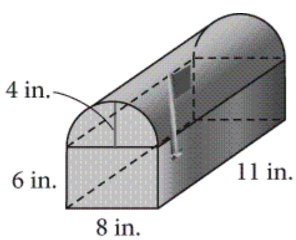
③ Find volume of all 4 trees:

$4(1500\pi) = 6000\pi$

④ Subtract volume of 4 trees from volume of box

$24000 - 6000\pi \approx \boxed{5{,}150 \text{ cm}^3 \text{ of empty space}}$

5.) How much paint, in square inches, is needed to paint the outside of the mailbox?



① Find surface area of rectangular prism w/o top:

2 ▭'s $6 \times 8 = 2(6)(8) = 96$ (front + back)

2 ▭'s $6 \times 11 = 2(6)(11) = 132$ (left + right)

1 ▭ $8 \times 11 = 88$ (bottom)

Total: $96 + 132 + 88 = 316 \text{ in}^2$

② Find surface area of half the cylinder:

$SA = \frac{2\pi r^2 + 2\pi r h}{2} = \pi r^2 + \pi r h$

$= \pi(4)^2 + \pi(4)(11)$

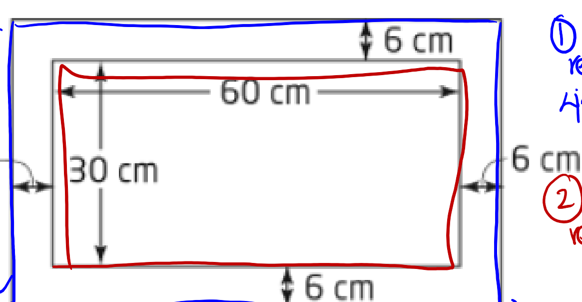
$= 16\pi + 44\pi$

$= 60\pi \text{ in}^2$

③ Find total:

$316 + 60\pi \approx \boxed{504 \text{ in}^2}$

6.) A picture measures 60 cm by 30 cm. The mat around the picture is 6 cm wide. Find the area of the mat only.



$30 + 6 + 6 = 42 \text{ cm}$

$60 + 6 + 6 = 72 \text{ cm}$

① Find area of large rectangle:

$42 \times 72 = 3024 \text{ cm}^2$

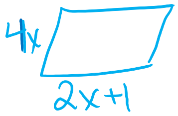
② Find area of smaller rectangle:

$30 \times 60 = 1800 \text{ cm}^2$

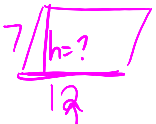
③ Subtract 2 areas:

$3024 - 1800 = \boxed{1224 \text{ cm}^2}$

7.) A parallelogram has a base of $4x$ and sides of $2x + 1$. The perimeter is 38 cm and the area is 60 cm^2. Find the height.



$4x + 4x + 2x + 1 + 2x + 1 = 38$
$12x + 2 = 38$
$12x = 36$
$x = 3$



$4(3) = 12$
$2(3) + 1 = 7$

$A = b \cdot h$
$\frac{60}{12} = \frac{12h}{12}$
$\boxed{h = 5 \text{ cm}}$

8.) The volume of a cube is 9,261cm^3

a) What are the dimensions of the cube?

$V = x^3$
$\sqrt[3]{9261} = \sqrt[3]{x^3}$
$21 = x$

$\boxed{21 \text{ cm} \times 21 \text{ cm} \times 21 \text{ cm}}$

b) What is the surface area of the cube?

$SA = 6x^2$
$= 6(21)^2$
$= \boxed{2{,}646 \text{ cm}^2}$

c) How would the volume of a rectangular prism with the same Length, width and height compare to the cube above?

It would be the same (9,261 cm^3)

d) How would the volume of square base pyramid with the same base and height compare to the cube above?

It would be $\frac{1}{3}$ the size ($9261 \div 3 = 3{,}087 \text{ cm}^3$)

9.) Suppose the radius of a sphere is quadrupled. What will happen to the surface area of the sphere? What will happen to the volume of the sphere?

$V = \frac{4}{3}\pi r^3$ vs. $V = \frac{4}{3}\pi(4r)^3 = \frac{4}{3}\pi(64r^3)$
$= \frac{256\pi r^3}{3}$

Divide: $\frac{256}{4} = 64$

$\boxed{\text{Volume is } 64 \text{ x's larger}}$

$SA = 4\pi r^2$
vs.
$SA = 4\pi(4r)^2 = 4\pi(16r^2) = 64\pi r^2$

Divide: $\frac{64}{4} = 16$

$\boxed{\text{SA is 16 x's larger}}$

10.) A triangular prism has a base that is a right triangle with shorter sides that measure 6 cm and 8 cm. The height of the prism is 10 cm.

a.) Predict how doubling the height affects the volume of the prism.

Answers vary

b.) Check your prediction by calculating the volume of the original prism and the volume of the new prism.

$V_1 = l_1 \cdot w_1 \cdot h_1 \div 2$
$= (6)(8)(10) \div 2$
$= 240 \text{ cm}^3$

$V_2 = l_2 \cdot w_2 \cdot h_2$
$= (6)(8)(20) \div 2$
$= 480 \text{ cm}^3$

$\frac{480}{240} = 2$

$\boxed{\therefore \text{The volume would double.}}$

c.) Was your prediction accurate?

Answers vary

d.) Is this true in general? If so, summarize the result.

yes, if the height doubles, but the other dimensions stay the same, the volume will double.