Review for Test #9 - Exponential Functions

Name ______________________________ Class ____________

1.) The population of a small grows at a rate of 2.1% annually. If the population is 450,000 in 2015, what will the population be in 2021? → $t = 2021 - 2015 = 6$

$P(t) = 450000(1+0.021)^t \Rightarrow P(t) = 450{,}000(1.021)^t$

$P(6) = 450000(1.021)^6 \approx \boxed{509761}$

2.) The breakdown of a sample of a chemical compound is represented by the function $s(t) = 700(0.3)^t$, where $s(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $s(t)$, explain what 0.3 and 700 represent.

700 – initially 700 mg of substance $s(0) = 700$

0.3 – growth (decay) factor – Each year there is 30% of the previous years substance.

3.) If you invest \$5000 in a bank that compounds interest annually, how much will you have in your account after 8 years? 6%

$f(t) = 5000(1+.06)^t = f(t) = 5000(1.06)^t$

$f(8) = 5000(1.06)^8 \approx \boxed{\$7969.24}$

4.) What is the common ratio in the geometric sequence $18, -6, 2, -\frac{2}{3}, \ldots$?

$\frac{-6}{18} = -\frac{1}{3}$ or $\frac{2}{-6} = -\frac{1}{3}$ or $\left(-\frac{2}{3}\right) \div 2 = -\frac{1}{3}$ $\boxed{r = -\frac{1}{3}}$

5.) Find the first four terms of the recursive sequence below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$n = 2$: $a_2 = a_1 - 2$, $a_2 = -3 - 2$, $a_2 = -5$

$n = 3$: $a_3 = a_2 - 3$, $a_3 = -5 - 3$, $a_3 = -8$

$n = 4$: $a_4 = a_3 - 4$, $a_4 = -8 - 4$, $a_4 = -12$

$\boxed{-3, -5, -8, -12}$

6.) The value of a car purchased for \$20,000 decreases at a rate of 12% per year. What will be the value of the car after 3 years?

(A) \$12,800.00
(B) \$13,629.44
(C) \$17,600.00
(D) \$28,098.56

$f(t) = 20000(1-.12)^t$

$f(t) = 20000(.88)^t$

$f(3) = 20000(.88)^3 \approx \$13629.44$

7.) A car depreciates (loses value) at a rate of 4.5% annually. Michael purchased a car for \$12,500. Which equation can be used to determine the value of the car, V, after 5 years?

(A) $V = 12{,}500(0.55)^5$
(B) $V = 12{,}500(0.955)^5$
(C) $V = 12{,}500(1.045)^5$
(D) $V = 12{,}500(1.45)^5$

$V = 12500(1-.045)^t$

$V = 12500(0.955)^t$

$V(5) = 12500(0.955)^5$

8.) Find the common ratio of the geometric sequence below:

$\frac{9x^3}{8y^2}, \frac{3y}{2x}, \frac{2y^4}{x^5}, \frac{8y^7}{3x^9}, \frac{32y^{10}}{9x^{13}}, \ldots$

$\left(\frac{3y}{2x}\right) \div \left(\frac{9x^3}{8y^2}\right) \rightarrow \frac{3y}{2x} \cdot \frac{8y^2}{9x^3} = \frac{24y^3 \div 6}{18x^4 \div 6} = \boxed{\frac{4y^3}{3x^4}}$

$\left(\frac{8y^7}{3x^9}\right) \div \left(\frac{2y^4}{x^5}\right)$

$\frac{8y^7}{3x^9} \cdot \frac{x^5}{2y^4} = \frac{8x^5y^7}{6x^9y^4} = \boxed{\frac{4y^3}{3x^4}}$

9.) Find the common ratio of the geomtric sequence below:

$\frac{7a^4}{8b^5}, \frac{a^3}{4b^3}, \frac{a^2}{14b}, \frac{ab}{49}, \frac{2b^3}{343}, \ldots$

$\left(\frac{a^3}{4b^3}\right) \div \left(\frac{7a^4}{8b^5}\right) \rightarrow \frac{a^3}{4b^3} \cdot \frac{8b^5}{7a^4} = \frac{8a^3b^5}{28a^4b^3} = \boxed{\frac{2b^2}{7a}}$

$\left(\frac{a^2}{14b}\right) \div \left(\frac{a^3}{4b^3}\right)$

$\frac{a^2}{14b} \cdot \frac{4b^3}{a^3} = \frac{4a^2b^3}{14a^3b} = \boxed{\frac{2b^2}{7a}}$

For 10 – 14, fill in the missing information in the chart.

10.)

Sequence	Arithmetic or Geometric	Explicit Formula
$a_1 = 35 \quad a_0 = 38$ 35, 32, 29, 26, … $d = -3$	arithmetic	$a_n = a_1 + d(n-1)$ $a_n = 35 - 3(n-1)$ $a_n = 35 - 3n + 3$ $\boxed{a_n = 38 - 3n}$ $a_n = a_0 + dr$ $\boxed{a_n = 38 - 3n}$
Recursive Formula	**10th Term**	
$a_{n+1} = a_n - 3$ for $a_1 = 35$	$a_{10} = 38 - 3(10)$ $a_{10} = 38 - 30 = \boxed{8}$	

11.)

Sequence	Arithmetic or Geometric	Explicit Formula
3, 6, 12, 24, 48, …	geometric	$a_n = a_1 \cdot r^{n-1}$ $a_n = 3 \bullet 2^{n-1}$ $a_1 = 3, \ r = 2$
Recursive Formula	**7th Term**	
$a_n = a_{n-1} \cdot 2$ for $a_1 = 3$	$a_7 = 3 \cdot 2^{7-1}$ $a_7 = 3 \cdot 2^6$ $\boxed{a_7 = 192}$	

$a_1 = 3 \cdot 2^0 = 3 \cdot 1 = 3$

$a_2 = 3 \cdot 2^1 = 3 \cdot 2 = 6$

$a_3 = 3 \cdot 2^2 = 3 \cdot 4 = 12$

12.) If $a_1 = -1.6$, then $a_0 = -1.6 - 2.7 = -4.3$

Sequence	Arithmetic or Geometric	Explicit Formula
$\boxed{-1.6, 1.1, 3.8, 6.5, \ldots}$	$\boxed{\text{arithmetic}}$	$a_n = a_1 + d(n-1)$ $a_n = -1.6 + 2.7(n-1)$ $a_n = -1.6 + 2.7n - 2.7$ $\boxed{a_n = 2.7n - 4.3}$
Recursive Formula	**11th Term**	OR
$d = 2.7$ $a_{n+1} = a_n + 2.7$ for $a_1 = -1.6$	$a_{11} = -4.3 + 2.7(11)$ $\boxed{a_{11} = -127.71}$	$a_n = a_0 + dn$ $\boxed{a_n = -4.3 + 2.7n}$

$a_2 = -1.6 + 2.7 = 1.1$
$a_3 = 1.1 + 2.7 = 3.8$

13.) If $a_1 = -2$, then $a_0 = -2 \cdot -2 = 4$

Sequence	Arithmetic or Geometric	Explicit Formula
-2, 1, -½, ¼, ... $r = -1/2$	geometric	$a_n = a_1 \cdot r^{n-1}$ $\boxed{a_n = -2 \cdot (-\frac{1}{2})^{n-1}}$ OR
Recursive Formula	**9th Term**	
$a_{n+1} = a_n \cdot -\frac{1}{2}$ for $a_1 = -2$	$a_9 = -2(-\frac{1}{2})^8 = \boxed{\frac{-1}{128}}$ OR $a_9 = 4(\frac{-1}{2})^9 = \boxed{\frac{-1}{128}}$	$a_n = a_0 \cdot r^n$ $\boxed{a_n = 4(-\frac{1}{2})^n}$

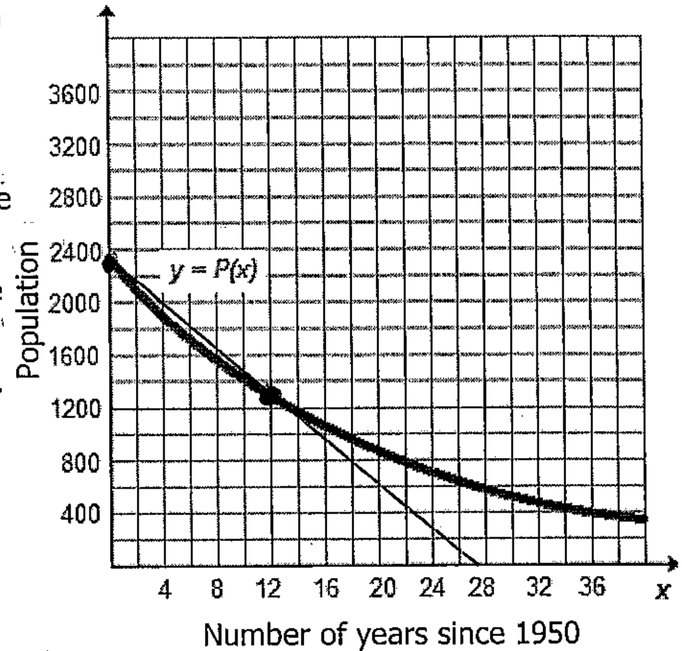
14.) If $f(1) = 5$, then $f(0) = 5 \cdot \frac{1}{4} = \frac{5}{4}$

Sequence	Arithmetic or Geometric	Explicit Formula
5, 20, 80, 320, ...	geometric	$f(n) = f(1) \cdot r^{n-1}$ $\boxed{f(n) = 5 \cdot (4)^{n-1}}$ OR
Recursive Formula	**6th Term**	
$r = 4$ $f(n) = 4 \bullet f(n-1)$ for $f(1) = 5$	$f(6) = 5 \cdot (4)^5 = \boxed{5120}$ OR $f(6) = \frac{5}{4}(4)^6 = \boxed{5120}$	$f(n) = f(0) \cdot r^n$ $\boxed{f(n) = \frac{5}{4}(4)^n}$

$f(2) = 4 \cdot f(1) = 4 \cdot 5 = 20$
$f(3) = 4 \cdot f(2) = 4 \cdot 20 = 80$

15.) The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2350. In 1962 the population was 1270. They chose an exponential decay model and arrived at the function: $P(x) = 2350(0.95)^x, x \geq 0$, where x is the number of years since 1950. The graph of this function is given below.

(0,2350)
(12,1270)



a. What is the y-intercept of the graph? Interpret its meaning in the context of the problem.

(0, 2350) The population was initially 2,350 people

b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?

It's not increasing - the population never goes up

c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

$[0, \infty)$

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

d. Write the linear function that this second group of scientists would have used.

(0,2350)
(12,1270)

① $m = \frac{1270-2350}{12-0} = \frac{-1080}{12} = -90$

② $y = -90x + b$

$b = 2350$

(b/c y-int happens when $x=0$)

$\boxed{L(x) = -90x + 2350}$

e. Graph the function on the coordinate plane.

✓

f. What is the x-intercept of the function? Interpret its meaning in the context of the problem.

x-int is when y (or $L(x)$) $= 0$.

$0 = -90x + 2350$

$90x = 2350$

$x \approx 26.1$ after ~26 yrs the population is 0