

Review for Test #11 - Quadratic Functions

Name _____

Class _____

1.) A baseball player throws a ball from the outfield toward home plate. The ball's height above the ground is modeled by the equation $y = -16x^2 + 48x + 6$, where y represents height, in feet, and x represents time, in seconds. The ball is initially thrown from a height of 6 feet.

a.) How many seconds after the ball is thrown will it again be 6 feet above the ground?

$$y=6$$

$$6 = -16x^2 + 48x + 6$$

$$0 = -16x^2 + 48x$$

$$0 = -16x(x-3)$$

$$\{0, 3\}$$

3 seconds

b.) What is the maximum height, in feet, that the ball reaches? 42 ft

$$y = -16(x^2 - 3x) + 6 \rightarrow \left(-\frac{-3}{2}\right)^2 = (-1.5)^2 = 2.25$$

$$y = -16(x^2 - 3x + 2.25) + 6 + 36$$

$$y = -16(x - 1.5)^2 + 42 \rightarrow \text{vertex: } (1.5, 42)$$

OR $x = \frac{-b}{2a} = \frac{-48}{2(-16)} = \frac{-48}{-32} = 1.5$

$$y = -16(1.5)^2 + 48(1.5) + 6 = 42$$

c.) How many seconds after the ball is thrown will it hit the ground? Round to the nearest tenth.

$$0 = -16x^2 + 48x + 6$$

$$x = \frac{-48 \pm \sqrt{(48)^2 - 4(-16)(6)}}{2(-16)} \rightarrow x = \frac{-48 \pm \sqrt{2688}}{-32}$$

$$x = -0.1 \text{ or } x = 3.1$$

3.1 sec

d.) Find the average rate of change from 0 to 1 second and explain the meaning in the context of the problem.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1) - f(0)}{1 - 0}$$

$$= \frac{38 - 6}{1 - 0} = \frac{32}{1}$$

$$x=1 \quad y = -16(1)^2 + 48(1) + 6 = 38$$

$$x=0 \quad y = -16(0)^2 + 48(0) + 6 = 6$$

2.) Consider the function $f(x) = -(x+3)^2 + 25$.

a.) Determine the vertex of the function.

Vertex: $(-3, 25)$

The ball is traveling at a rate of 32 ft/s from 0 to 1 sec.

b.) Find the domain and range of the function.

Domain: all reals
Range: $f(x) \leq 25$

c.) Find the roots of the function.

$$0 = -(x+3)^2 + 25$$

$$-25 = -(x+3)^2$$

$$25 = (x+3)^2$$

$$\pm 5 = x+3$$

$$-3 \pm 5 = x$$

{-2, -8}

- 3.) An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t , and the height of the arrow in meters, h , is given by:

$$h(t) = -4.9t^2 + 29.4t + 2.5$$

- a.) Complete the square for this function.

$$h(t) = -4.9(t^2 - 6t + 9) + 2.5 + 44.1$$

$$h(t) = -4.9(t-3)^2 + 46.6$$

- b.) What is the maximum height of the arrow? Explain how you know.

vertex is $(3, 46.6) \rightarrow$ The maximum height is 46.6 ft.

- c.) How long does it take the arrow to reach its maximum height? Explain how you know.

3 seconds - vertex is $(3, 46.6)$

- d.) What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds?

$$\left. \begin{array}{l} h(2) = 41.7 \\ h(1) = 27 \end{array} \right\} \frac{h(2) - h(1)}{2 - 1} = \frac{41.7 - 27}{2 - 1} = \frac{14.7}{1} \quad \boxed{14.7 \text{ m/s}}$$

- e.) Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds and explain the difference in the context of the problem.

$$h(3) = 46.6 \quad \left\{ \frac{h(3) - h(2)}{3 - 2} = \frac{46.6 - 41.7}{3 - 2} = \frac{4.9}{1} \quad \boxed{4.9 \text{ m/s}} \right. \quad \begin{array}{l} \text{The arrow is} \\ \text{slowing down} \\ \text{due to gravity} \end{array}$$

- f.) How long does it take the arrow to hit the ground? Round to the nearest hundredth of a second. Show your work or explain.

$$0 = -4.9t^2 + 29.4t + 2.5 \quad t = \frac{-29.4 \pm \sqrt{(29.4)^2 - 4(-4.9)(2.5)}}{2(-4.9)}$$

- g.) What does the constant term (c) in the original equation tell you about the arrow?

It tells us the height the arrow is when it is first shot.

- h.) What does the first-degree coefficient (a) tell you about the arrow's flight?

It is shot up then falls down

- 4.) Given the quadratic function $f(x) = 4x^2 + 4x + 5$, answer the following questions:

- a.) Find the vertex of the function by using the equation for the axis of symmetry.

$$x = \frac{-b}{2a} = \frac{-4}{2(4)} = \frac{-4}{8} = -0.5 \quad \begin{array}{l} f(-0.5) = 4(-0.5)^2 + 4(-0.5) + 5 \\ f(-0.5) = 4 \end{array} \quad \boxed{(-0.5, 4)}$$

- b.) Find the vertex of the function by completing the square to put the function in vertex form.

$$f(x) = 4(x^2 + x) + 5$$

$$f(x) = 4(x^2 + x + \frac{1}{4}) + 5 - 1$$

$$f(x) = 4(x + \frac{1}{2})^2 + 4$$

$$\boxed{(-0.5, 4)}$$

- c.) Find the domain and the range of the function.

$$f(x) \geq 4$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

5.) A parabola intersects the x-axis at $x = -3$ and $x = 2$ and intersects the y-axis at $y = -18$. Which is an equation of the parabola?

(A) $y = 3x^2 + 3x - 18$

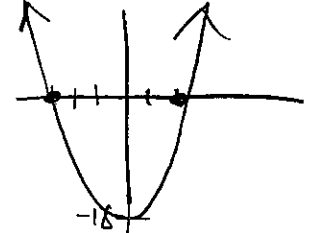
(B) $y = -3x^2 - 3x - 18$

$a > 1$ (opens up)

(C) $y = -18x^2 - 1.5x - 1$

(D) $y = 18x^2 + 1.5x + 1$

$c \neq -18$



6.) How many real roots/zeros do the following functions have? Explain your answer.

a.) $f(x) = 2x^2 + 5x + 7$

$D = b^2 - 4ac$

$D = (5)^2 - 4(2)(7)$

$D = 25 - 56 = -29$

No real zeros because $D < 0$ (can't square root a neg. # w/ real values)

b.) $g(x) = 4x^2 - 28x + 49$

$D = (-28)^2 - 4(4)(49)$

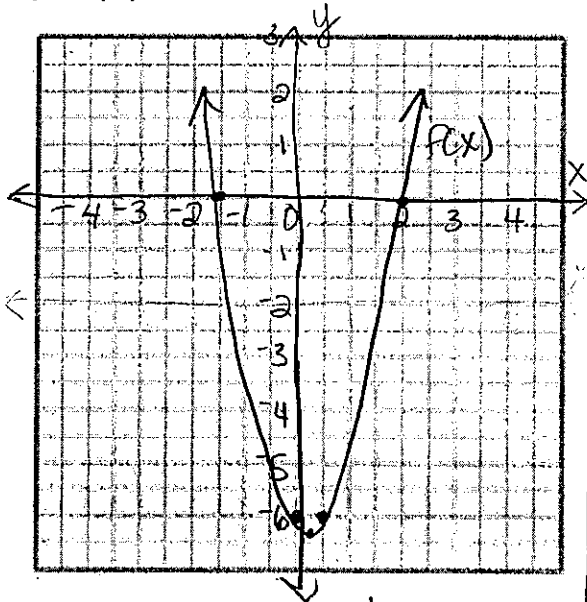
$D = 784 - 784$

$D = 0$

one real solution because $D = 0$

7.) Graph the following quadratic functions by identifying the key features. If necessary, round to the nearest tenth.

a.) $f(x) = 2x^2 + x - 6$



$f(x) = (2x + 3)(x - 2)$

Roots: $0 = (2x + 3)(x - 2)$

$\{-3/2, 2\}$

y-int:

$f(0) = 2(0)^2 + 0 - 6 = -6$

Axis of symmetry:

$x = -\frac{b}{2a} = -\frac{1}{2(2)} = -\frac{1}{4}$

vertex: $f(-1/4) = -\frac{49}{8} = -6.125$

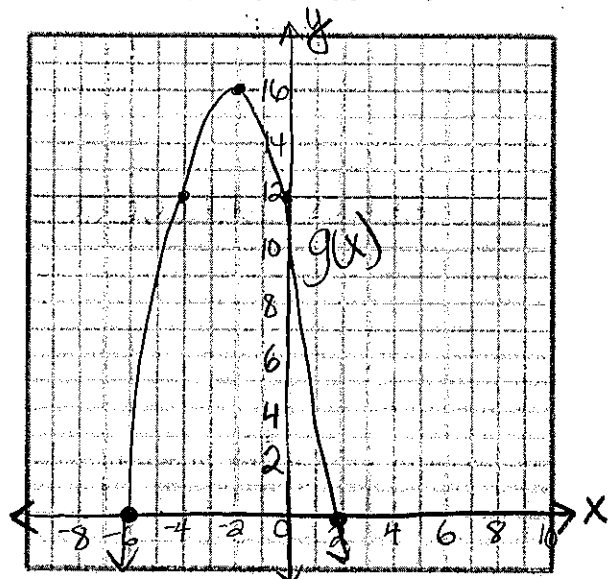
Roots: $\{-3/2, 2\}$

y-int: $(0, -6)$

Axis of sym: $x = -0.25$

vertex: $(-0.25, -6.125)$

b.) $g(x) = -(x + 6)(x - 2)$



Roots: $0 = -(x + 6)(x - 2)$

Roots: $\{-6, 2\}$

y-int: $g(0) = 12$

Axis of sym: $x = -\frac{6+2}{2} = -2$

$g(-2) = 16$

vertex: $(-2, 16)$

Roots: $\{-6, 2\}$

y-int: $(0, 12)$

Axis of sym: $x = -2$

vertex: $(-2, 16)$

