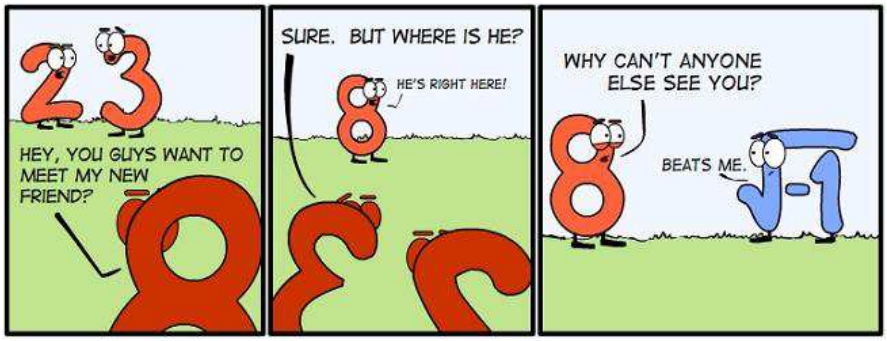
Unit 10 Notes

Quadratic Equations and Functions



Tentative Schedule

Day	Class Work	Assignment
Thurs. 3/5	Test #9	Factoring Review
Fri. 3/6 Mon. 3/9	Discuss Zero Product Rule Review Simplifying Radicals	Video #10.1 – Solving Quadratic Equations by Factoring
Tues. 3/10	PS #10.1	Video #10.2 – Solving Quadratic Equations by Square Rooting
Wed. 3/11 Thurs. 3/12	PS #10.2	Video #10.3 – Completing the Square Day 1
Fri. 3/13	PS #10.3	Video #10.4 – Completing the Square Day 2
Mon. 3/16 Tues. 3/17	PS #10.4	Video #10.5 – Solving Quadratic Equations by Completing the Square
Wed. 3/18	PS #10.5	Video #10.6 – Solving Quadratic Equations by the Quadratic Formula
Thurs. 3/19 Fri. 3/20	PS #10.6	Video #10.7 – Applications of Quadratic Equations
Mon. 3/23	PS #10.7 Take-Home Quiz Due	Finish and Correct Practice Packet
Tues. 3/24 Wed. 3/25	Review for Test #10	Review for Test #10
Thurs. 3/26	**Test #10**	Video #11.1

Name: _____

Notes 10.1 - Solving Quadratic Equations by Factoring

Find values of c and d that satisfy each of the following equations. (There may be more than one correct answer.)

1.) $cd = 0$

$c = 0$ or $d = 0$

2.) $(c-5)d = 2$

c and d can be anything

3.) $(c-5)d = 0$

$c = 5$ or $d = 0$

Zero Product Rule:

There are two numbers that multiply to zero. What do you know about these two numbers?

Zero Product Rule
If a and b are two numbers or two expressions and $a \cdot b = 0$, then: $a = 0$ or $b = 0$

Linear vs. Quadratic Equations

Linear Equations	Versus	**Quadratic Equations**
• Highest Exponent is 1		• Highest Exponent is 2
• Ex. $5x+2=3x+1$		• Ex. $x^2+7x+10=0$
• # of possible solutions: 1		• # of possible solutions: 2
• To Solve: get the variables on one side and the constants on the other side		• To Solve: • set equal to 0 • factor OR • complete the square OR • quadratic formula

Solving quadratic equations:

4.) $x^2 - 7x + 10 = 0$

$(x-5)(x-2)=0$

$x-5=0$ | $x-2=0$

$+5\ +5$ | $+2\ +2$

$x=5$ | $x=2$

$\{5,2\}$

5.) $(x+5)(x-8)=30$

$x^2-8x+5x-40=30$

$x^2-3x-40=30$

$-30\ -30$

$x^2-3x-70=0$

$(x-10)(x+7)=0$

$x-10=0$ | $x+7=0$

$x=10$ | $x=-7$

$\{-7,10\}$

6.) $m^2 = 7m$

$-7m\ -7m$

$m^2-7m=0$

$m(m-7)=0$

$m=0$ | $m-7=0$

$m=7$

$\{0,7\}$

7.) $-6y^3 - 12y = -27y^2$

$+27y^2\ \ +27y^2$

$-6y^3+27y^2-12y=0$

$-3y(2y^2-9y+4)=0$

$-3y(2y-1)(y-4)=0$

$-3y=0$ | $2y-1=0$ | $y-4=0$

$y=0$ | $2y=1$ | $y=4$

$y=\frac{1}{2}$

$\{0,\frac{1}{2},4\}$

8.) If the solution set to a quadratic equation is $\{1,-4\}$, what is the equation?

If $x=1$, then $x-1=0$.

If $x=-4$, then $x+4=0$

$(x-1)(x+4)=0$

$x^2+4x-x-4=0$

$x^2+3x-4=0$

★Isolate the perfect square

Notes 10.2 - Solving Quadratic Equations by Square Rooting

Solve the following quadratic equations.

1.) $\frac{2(x-3)^2}{2} = \frac{48}{2}$

$\sqrt{(x-3)^2} = \sqrt{24}$

$x-3 = \pm\sqrt{24}$

$x-3 = \pm\sqrt{4}\sqrt{6}$

$x-3 = \pm 2\sqrt{6}$

$+3 \quad +3$

$x = 3 \pm 2\sqrt{6}$

2.) $\frac{3}{27} = \frac{27(x+1)^2}{27}$

$\sqrt{\frac{1}{9}} = \sqrt{(x+1)^2}$

$\pm\frac{1}{3} = x+1$

$-1 \quad -1$

$-1 \pm \frac{1}{3} = x$

$-1+\frac{1}{3} = -\frac{2}{3} \qquad -1-\frac{1}{3} = -\frac{4}{3}$

$\left\{-\frac{4}{3}, -\frac{2}{3}\right\}$

3.) $\frac{3x}{x+2} = \frac{4}{x-2}$ ★cross multiply!

$3x(x-2) = 4(x+2)$

$3x^2-6x = 4x+8$

$-4x-8 \quad -4x-8$

$3x^2-10x-8=0$

$(3x+2)(x-4)=0$

$3x+2=0$	$x-4=0$
$3x=-2$	$x=4$
$x=-\frac{2}{3}$	

$\left\{-\frac{2}{3}, 4\right\}$

4.) $\frac{x+5}{3} = \frac{10}{x-8}$

$(x+5)(x-8) = (10)(3)$

$x^2-8x+5x-40=30$

$x^2-3x-40=30$

$-30 \quad -30$

$x^2-3x-70=0$

$(x-10)(x+7)=0$

$x-10=0$	$x+7=0$
$x=10$	$x=-7$

$\{-7, 10\}$

Notes 10.3 - Completing the Square Day 1

1.) Write the following standard form quadratic expressions as perfect squares.

$x^2+12x+36$ $\frac{12}{2}=6$ $\sqrt{36}=\pm 6$	$(x+6)(x+6)$ $6x$ $6x$ $(x+6)^2$
$x^2-12x+36$ $\frac{-12}{2}=-6$ $\sqrt{36}=\pm 6$	$(x-6)^2$
$x^2+20x+100$ $\frac{20}{2}=10$ $\sqrt{100}=\pm 10$	$(x+10)^2$
$x^2-3x+\frac{9}{4}$ $\frac{-3}{2}$ $\sqrt{\frac{9}{4}}=\pm\frac{3}{2}$	$(x-\frac{3}{2})^2$
$x^2+100x+2{,}500$ $\frac{100}{2}=50$ $\sqrt{2500}=\pm 50$	$(x+50)^2$
x^2+8x+3	Not factorable

$(\frac{b}{2})^2$!!!!

Find an expression equivalent to the expression below that includes a perfect square binomial.

1.) x^2+8x+3 $b=8$

$x^2+8x+16+3-16$ $(\frac{8}{2})^2=4^2=16$

perfect square — must subtract to keep expression same

$(x+4)^2-13$

2.) $x^2+9x+11$ $(\frac{9}{2})^2=\frac{81}{4}$

$x^2+9x+\frac{81}{4}+11-\frac{81}{4}$

$(x+\frac{9}{2})^2+\frac{37}{4}$

Notes 10.4 - Completing the Square Day 2

Complete the square for the examples below:

1.) $2x^2+16x+3$

$2(x^2+8x\ \ )+3$

$\left(\frac{8}{2}\right)^2=4^2=16$

$2(x^2+8x+16)+3-32$

$2(x+4)^2-29$

You're adding 32 because you must distribute the 2 to the 16.

2.) $4x^2-2x-5$

$4(x^2-\frac{1}{2}x\ \ )-5$

$\left(\frac{-\frac{1}{2}}{2}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}$

$4(x^2-\frac{1}{2}x+\frac{1}{16})-5-\frac{1}{4}$ ← $\left(\frac{1}{16}\cdot 4=\frac{1}{4}\right)$

$4(x-\frac{1}{4})^2-\frac{25}{4}$

3.) A certain business is marketing their product and has collected data on sales and prices for the past few years. They determined that when they raised the selling price of the product, the number of sales went down. The cost of producing single item is $10.

Selling Price (s)	Quantity Sold (q)
10	1,000
15	900
20	800
25	700

a.) Using the data they collected in this table, determine a linear expression to represent the quantity sold, *q*.

b.) Now find an expression to represent the profit function, *P*.

Notes 10.5 - Solving Quadratic Equations by Completing the Square

Solve the following equations for x.

1.) $12 = x^2 + 6x$ (in simplest radical form)

2.) $4x^2 + 93 = 40x$ (round to the nearest tenth)

① Make sure $a=1$ and ax^2+bx are isolated on one side.

$12 = x^2 + 6x$

$\frac{4x^2}{4} - \frac{40x}{4} = -\frac{93}{4}$

$x^2 - 10x = -\frac{93}{4}$

② Find the value that completes the square and add it to both sides of the equation.

$\left(\frac{6}{2}\right)^2 = 3^2 = 9$

$12 + 9 = x^2 + 6x + 9$

$21 = x^2 + 6x + 9$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$x^2 - 10x + 25 = -\frac{93}{4} + 25$

$x^2 - 10x + 25 = \frac{7}{4}$

③ Factor the perfect square expression.

$21 = x^2 + 6x + 9$

$21 = (x+3)^2$

$x^2 - 10x + 25 = \frac{7}{4}$

$(x-5)^2 = \frac{7}{4}$

④ Square root both sides of the equation

$\sqrt{21} = \sqrt{(x+3)^2}$

$\pm\sqrt{21} = x+3$

$\sqrt{(x-5)^2} = \sqrt{\frac{7}{4}}$

$x - 5 = \pm\frac{\sqrt{7}}{2}$

⑤ Isolate the variable

$\pm\sqrt{21} = x+3$

$-3 \quad -3$

$-3 \pm \sqrt{21} = x$

$+5 \quad +5$

$x = 5 \pm \frac{\sqrt{7}}{2}$

⑥ Write each answer out.

$x = -3+\sqrt{21}, \quad x = -3-\sqrt{21}$

$\{-3+\sqrt{21}, -3-\sqrt{21}\}$

$x = 5+\frac{\sqrt{7}}{2}, \quad x = 5-\frac{\sqrt{7}}{2}$

⑦ Round if necessary.

$x \approx 6.3$ or $x \approx 3.7$

$\{6.3, 3.7\}$

Lab Notes - Deriving the Quadratic Formula

Derive the quadratic formula by completing the square in the equation $ax^2+bx+c=0$.

$$ax^2+bx+c=0$$

$$-c \qquad -c$$

$$\frac{ax^2+bx}{a}=\frac{-c}{a}$$

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Complete the square: $\left(\frac{\frac{b}{a}}{2}\right)^2=\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

Rewrite as one fraction

$$\frac{4a}{4a}\cdot\frac{-c}{a}+\frac{b^2}{4a^2}$$

$$\frac{-4ac+b^2}{4a^2}=$$

$$\frac{b^2-4ac}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2-4ac}{4a^2}$$

Factor the perfect square trinomial.

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Square root

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

Isolate x.

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

QUADRATIC FORMULA

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Notes 10.6 - Solving Quadratic Equations Using the Quadratic Formula

What is the quadratic formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula to solve each equation.

1.) $\frac{1}{2}r^2 - 6r = 2$

$\cdot 2 \quad \cdot 2$

$r^2 - 12r + 36 = 4 + 36$

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$\sqrt{(r-6)^2} = \sqrt{40}$

$r - 6 = \pm\sqrt{40}$

$r - 6 = \pm\sqrt{4}\sqrt{10}$

$r - 6 = \pm 2\sqrt{10}$

$r = 6 \pm 2\sqrt{10}$

$\{6+2\sqrt{10}, 6-2\sqrt{10}\}$

2.) $2y^2 + 3y - 5 = 4$

$+5 \quad +5$

$\frac{2y^2}{2} + \frac{3y}{2} = \frac{9}{2}$

$y^2 + \frac{3}{2}y + \frac{9}{16} = \frac{9}{2} + \frac{9}{16}$

$\left(\frac{\frac{3}{2}}{2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$\sqrt{\left(y + \frac{3}{4}\right)^2} = \sqrt{\frac{81}{16}}$

$y + \frac{3}{4} = \pm\frac{9}{4}$

$-\frac{3}{4} \quad -\frac{3}{4}$

$y = -\frac{3 \pm 9}{4}$

$-\frac{3+9}{4} = \frac{6}{4} = \frac{3}{2}$ $\qquad -\frac{3-9}{4} = -\frac{12}{4} = -3$

$\left\{\frac{3}{2}, -3\right\}$

Notes 10.7 - Applications of Quadratic Equations

1.) The length of a rectangle is sixteen more than twice the width. If the area of the rectangle is 40 square inches, find the dimensions of the rectangle. Only an algebraic solution is acceptable.

$L \cdot W = A$

$(2x+16)x = 40$

$2x^2 + 16x = 40$

$2x^2 + 16x - 40 = 0$

$2(x^2 + 8x - 20) = 0$

$2(x+10)(x-2) = 0$

$x = -10$ OMIT $\quad x = 2$

$x = 2$

The length is 20 inches and the width is 2 inches.

Reminder:

Consecutive Integers

5	a
6	a+1
7	a+2

Consecutive Even Integers

4	b
6	b+2
8	b+4

Consecutive Odd Integers

5	c
7	c+2
9	c+4

2.) Find three consecutive positive integers such that the product of the second and third, added to the first equals 47.

Let 1st int = x
Let 2nd int = x+1
Let 3rd int = x+2

$(x+1)\cdot(x+2)+x=47$ (second · third + added to first equals 47)

$x^2+2x+x+2+x=47$

$x^2+4x+2=47$

$-47\ \ -47$

$x^2+4x-45=0$

$(x+9)(x-5)=0$

$x=-9$ or $x=5$

$x=-9$ omit (must be pos.)

The three integers are 5, 6, and 7.

3.) If the ratio of the width to the length of a rectangle is 3:5 and the area of the rectangle is 735, find the length and the width of the rectangle.

Let width = 3x
Let length = 5x

$(3x)(5x)=735$

$\frac{15x^2}{15}=\frac{735}{15}$

$\sqrt{x^2}=\sqrt{49}$

$x=\pm 7$

omit −7 b/c length/width can't be negative

width $= 3x = 3(7) = 21$

length $= 5x = 5(7) = 35$

The width is 21 units and the length is 35 units.