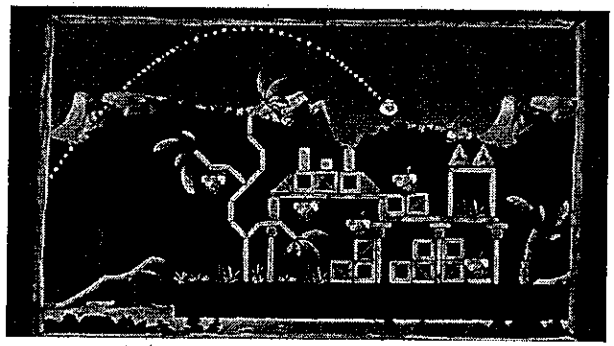
Unit 11 Notes
Quadratic Equations and Functions



Tentative Schedule

Day	Class Work	Assignment
Tues. 4/7 (all)	Exploring the Symmetry in Graphs of Quadratic Functions	P.S. #11.1
Wed. 4/8 (S) Thurs. 4/9 (R)	Graphing Quadratic Functions from Factored Form	P.S. #11.2
Fri. 4/10 (all)	Graphing Quadratic Functions from the Vertex Form	P.S. #11.3
Mon. 4/13 (S) Tues. 4/14 (R)	Graphing Quadratic Functions from the Standard Form	P.S. #11.4
Wed. 4/15 (all)	Applications of Quadratic Functions	P.S. #11.5
Wed. 4/15 (R) Thurs. 4/16 (S)	Lab: Discriminant	
Thurs. 4/16 (S) Fri. 4/17 (R)	Review for Test #11	Review
Mon. 4/20 (all)	**Test #11**	Begin Transformations

Name: ______________________________

Quadratic Function Summary

Key Features	Factored Form	Vertex Form	Standard Form
Form	$f(x)=a(x-m)(x-n)$	$f(x)=a(x-h)^2+k$	$f(x)=ax^2+bx+c$
Axis of symmetry	Put in vertex form by completing the square or put in standard form by distributing. Follow rules for new forms.	$x=h$	$x=-\frac{b}{2a}$
Vertex	Put in vertex form by completing the square or put in standard form by distributing. Follow rules for new forms.	(h,k)	$\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)$
Opens up/down	If $a>0$, parabola opens up (making vertex a minimum). If $a<0$, parabola opens down (making vertex a maximum).		
Y-intercept	$(0,f(0))$	$(0,f(0))$	$(0,c)$
Zeros/Roots	Set each factor equal to 0. $x=m$, $x=n$	Set $f(x)=0$ and solve by isolating the perfect square binomial and square rooting.	Set $f(x)=0$ and solve by factoring, completing the square, or the quadratic formula.
Two real zeros if	$m\neq n$	k and a have opposite signs	$b^2-4ac>0$
One real zero if	$m=n$	$k=0$	$b^2-4ac=0$
No real zeros if	A quadratic function with no real zeros cannot be written in factored form with real coefficients	k and a have same signs	$b^2-4ac<0$
Range	If $a>0$ and the vertex occurs at (h,k), range is $f(x)\geq k$. If $a<0$ and the vertex occurs at (h,k), range is $f(x)\leq k$.		

Notes 11.1 - Exploring the Symmetry in Graphs of Quadratic Functions

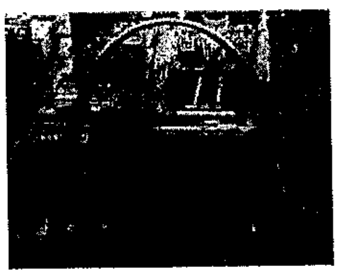
Axis of symmetry A vertical line that makes two sides of a graph look like mirror images of each other.	*Vertex* The turning point of a parabola (minimum or maximum – which is determined by a value)
Y-intercept The point at which a graph intersects the *y*-axis. (x always equals 0)	*Roots/Zeros* A solution to an equation of the form $f(x)=0$. (Similar to *x*-intercept)
Domain The set of *x*-values for which a function is defined.	*Range* The set of *y*-values for which a function is defined.

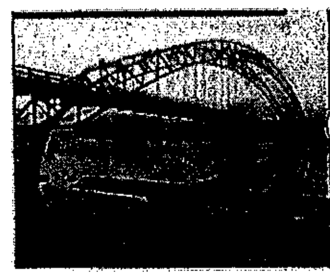
Below are some examples of curves found in architecture around the world. Some of these might be represented by graphs of quadratic functions. What are the key features these curves would have in common with a graph of a quadratic function?

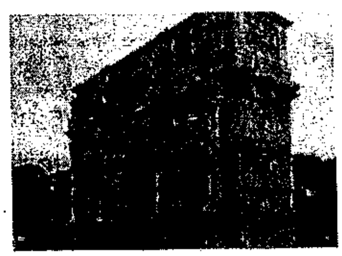
The photographs of architectural features above MIGHT be closely represented by graphs of quadratic functions. Answer the following questions based on the pictures.

1.) How would you describe the overall shape of a quadratic function?

- Curved


St. Louis Arch


Bellos Falls Arch Bridge

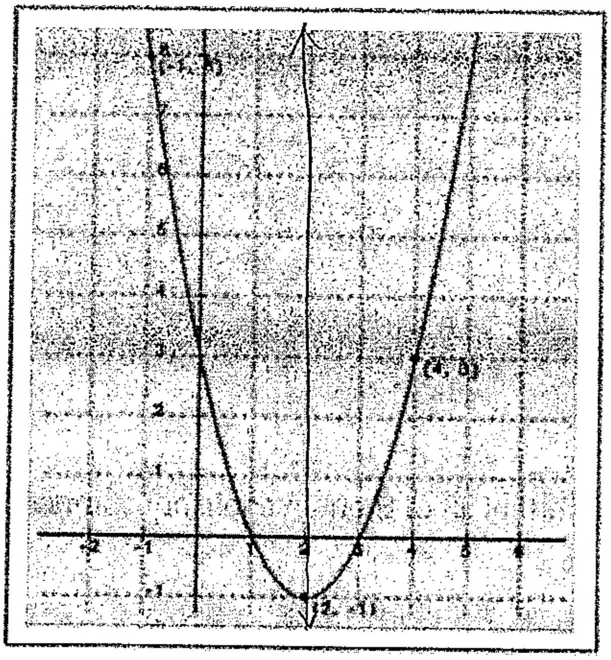

Arch of Constantine


Roman Aqueduct

2.) What is similar or different about the overall shape of the above curves?

Use the graphs of quadratic functions A and B to fill in the table answer the questions on the following page.

Graph *A*



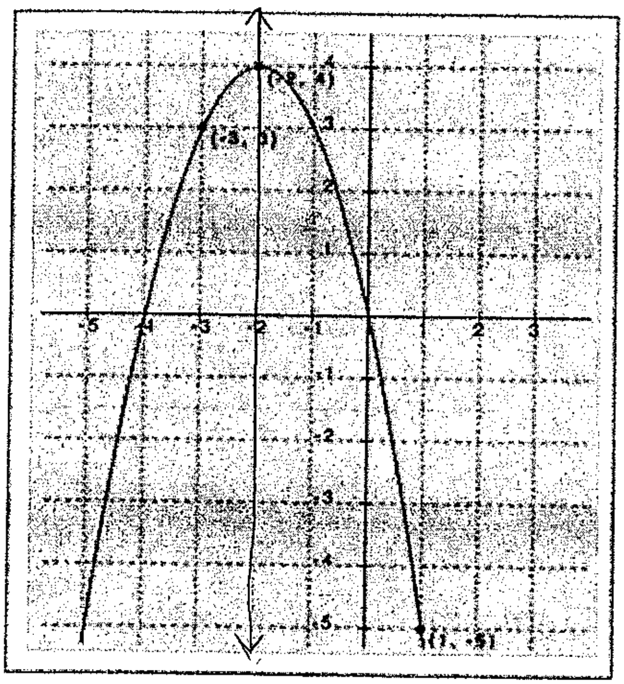
$f(x)= (x-2)^2-1$

$3=a(4-2)^2-1$

$3=4a-1$

x	$f(x)$
−1	8
0	3
1	0
2	−1
3	0
4	3
5	8

Graph *B*



$g(x)=-(x+2)^2+4$

$3=a(-1)^2+4$

$3=a+4$

$a=-1$

x	$f(x)$
-5	-5
-4	0
−3	3
−2	4
-1	3
0	0
1	−5

Use your graphs and table of values fill in the blanks or answer the questions for each below.

		Graph *A*	Graph *B*
1	*Axis of symmetry*	$x=2$	$x=-2$
2	*Vertex*	$(2,-1)$	$(-2,4)$
3	*Min or max?*	min	max

		Graph *A*	Graph *B*
4	*Y-intercept*	(0,3)	(0,0)
5	*X-intercepts*	(1,0) + (3,0)	(-4,0) + (0,0)
6	*Roots/Zeros*	{1,3}	{-4,0}
7	*Points of Symmetry*	Find $f(-1)$ and $f(5)$. f(-1)=8 f(5)=8 Is $f(7)$ greater than or less than 8? Explain. f(7)>8 because the graph is increasing when x>2 and f(5)=8	Find $f(-1)$ and $f(-3)$. f(-1)=3 F(-3)=3 We know that $f(-2)=-12$. Predict the value for $f(-6)$ and explain. f(-6)=-12 The graph is symmetrical
8	*Increasing and Decreasing Intervals*	On what intervals of the domain is the function increasing? (2,∞) On what intervals of the domain is the function decreasing? (-∞,2)	On what intervals of the domain is the function increasing? (-∞,-2) On what intervals of the domain is the function decreasing? (-2,∞)
9	*Domain and Range*	Determine the domain and range of the function. D: all ℝ R: f(x) ≥ -1	Determine the domain and range of the function. D: all ℝ R: f(x) ≤ 4

Understanding the symmetry of quadratic functions and their graphs (Look at row 7 in the chart and the tables).

3.) What patterns do you see in the tables of values you made next to Graph *A* and Graph *B*?

- x-values increase by 1
- y-values increase or decrease to vertex then repeat same pattern in opposite order
- points are symmetrical

Finding the vertex and axis of symmetry (Look at rows 1, 2, and 5 of the chart).

4.) How can we know the *x*-coordinates of the vertex by looking at the *x*-coordinates of the zeros (or any pair of symmetric points)?

it's halfway between (average of x-coordinates

Understanding end behavior (Look at row 3 of the chart)

5.) What happens to the *y*-values of the functions as the *x*-values increase to very large numbers? What about as the *x*-values decrease to very small numbers (in the negative direction)?

A: approach $+\infty$ in both directions
B: approach $-\infty$ in both directions

$-(x^2+4x+4)+4$
$-x^2-4x-4+c$

6.) How can we whether a graph of a quadratic function will open up or down?

A: $f(x)=(x-2)^2-1$
$f(x)=x^2-4x+3$
opens up
positive leading coefficient

B: $f(x)=-(x+2)^2+4$
$f(x)=-x^2-4x$
opens down - negative leading coefficient

Understanding intervals on which the function is increasing (or decreasing) (Look at row 8 in the chart).

7.) Is it possible to determine the exact intervals that a quadratic function is increasing or decreasing just by looking at a graph of the function?

If the vertex is clearly located

Finding a unique quadratic function.

8.) Can you graph a quadratic function if you don't know the vertex? Can you graph a quadratic function if you only know the *x*-intercepts?

no

9.) Remember that we need to know at least two points to define a unique line. Can you identify a unique quadratic function with just two points? Explain.

No - many quadratic functions pass through the same 2 points

10.) What is the minimum number of points you would need to identify a unique quadratic equation?

3

8

Notes 11.2 - Graphing Quadratic Functions from Factored Form

Warm-up – Solve the following quadratic equations.

1.) $x^2 + 6x - 40 = 0$

$(x+10)(x-4)=0$

$\{-10, 4\}$

2.) $2x^2 + 11x = x^2 - x - 32$

$x^2 + 12x + 32 = 0$

$(x+4)(x+8) = 0$

$\{-4, -8\}$

Consider the equation $y = x^2 + 6x - 40$.

3.) Given this quadratic equation, can you find the point(s) where the graph crosses the x-axis?

$0 = x^2 + 6x - 40$

$0 = (x+10)(x-4)$

$\{-10, 4\}$

4.) How can we write a corresponding quadratic equation if we are given a pair of roots?

$y = (x+10)(x-4)$

$y = x^2 + 6x - 40$

5.) In the last session, we learned about the symmetrical nature of the graph of a quadratic function. How can we use that information to find the vertex for the graph?

average the roots to find axis of symmetry

$x = \frac{-10+4}{2} = \frac{-6}{2}$ $\quad x = -3$

$y = (-3)^2 + 6(-3) - 40$

$y = 9 - 18 - 40$

$y = -49$

$\boxed{(-3, -49)}$

6.) How could we find the y-intercept?

Find y when $x = 0$.

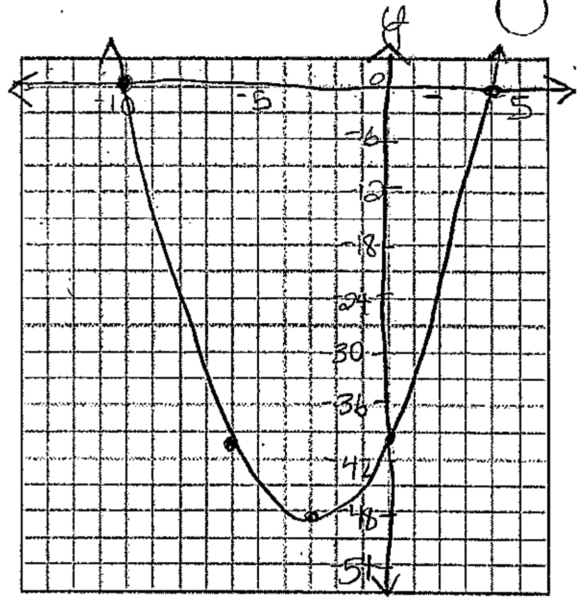
$y = 0^2 + 6(0) - 40$

$y = -40$

$\boxed{(0, -40)}$

7.) What else can we say about the graph based on our knowledge of the symmetrical nature of the graph of a quadratic function? Can we determine the coordinates of any other points?

Yes $(-6, -40)$



8.) Plot the points you know and connect them to show the graph of the equation.

9.) Graph the following function and identify key features of the graph.

$f(x) = -x^2 + 3x + 10$

$f(x) = -(x^2 - 3x - 10)$

$f(x) = -(x-5)(x+2)$

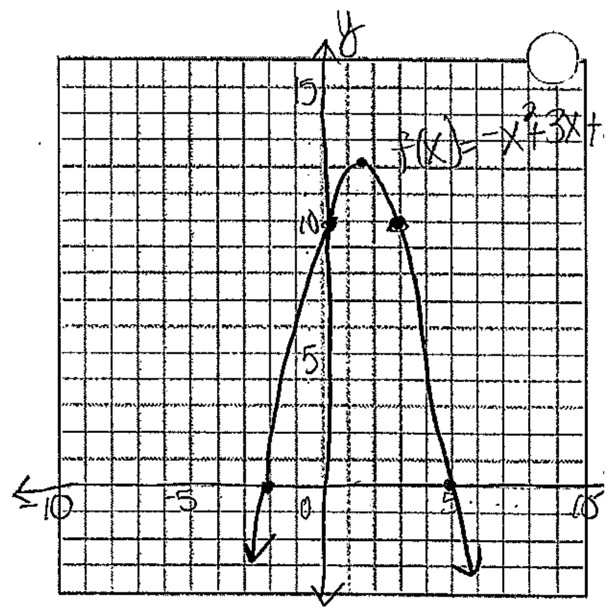
$0 = -(x-5)(x+2)$

$\boxed{\{5, -2\}}$ - roots

$x = \frac{5-2}{2} = \frac{3}{2} = 1.5$ Axis of sym $x = 1.5$

Vertex: $(1.5, 12.25)$ - max

y-int: $f(0) = 10$ $\boxed{(0,10)}$



new point of symmetry

$(3, 10)$

A science class designed a ball launcher and tested it by shooting a tennis ball straight up from the top of a 15-story building. They determined that the motion of the ball could be described by the function:

$$h(t) = -16t^2 + 144t + 160$$

10.) With a graph, we can see the number of seconds it takes for the ball to reach its peak, and also how long it takes to hit the ground. Factor the expression.

$h(t) = -16(t^2 - 9t - 10)$

$h(t) = -16(t - 10)(t + 1)$

11.) Once we have the function in factored form, what do we need to know in order to graph it?

$0 = -16(t-10)(t+1)$

roots – $\{10, -1\}$

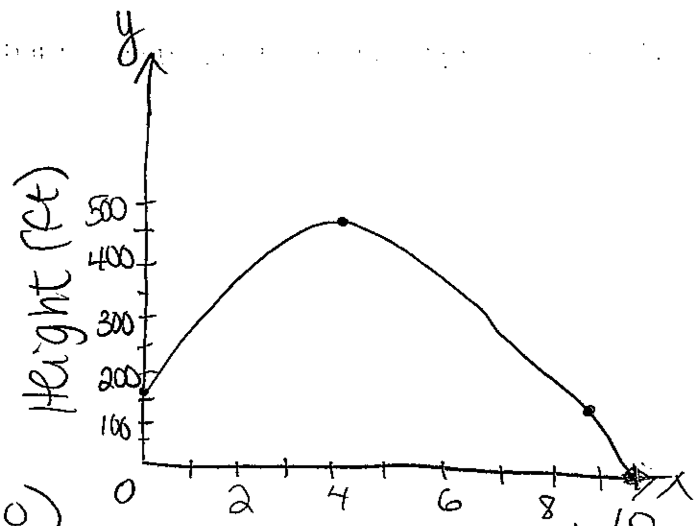
$x = \frac{9}{2} = 4.5$

Axis of sym:

$x = 4.5$

Vertex: $(4.5, 484)$ – max

y-int: $(0, 160)$ symmetry $(9, 160)$



Time (sec)

12.) Using the graph, at what time does the ball hit the ground?

10 sec

13.) Over what domain is the ball rising? Over what domain is the ball falling?

$(0, 4.5)$

$(4.5, 10)$

Factored Form of a Quadratic Function:

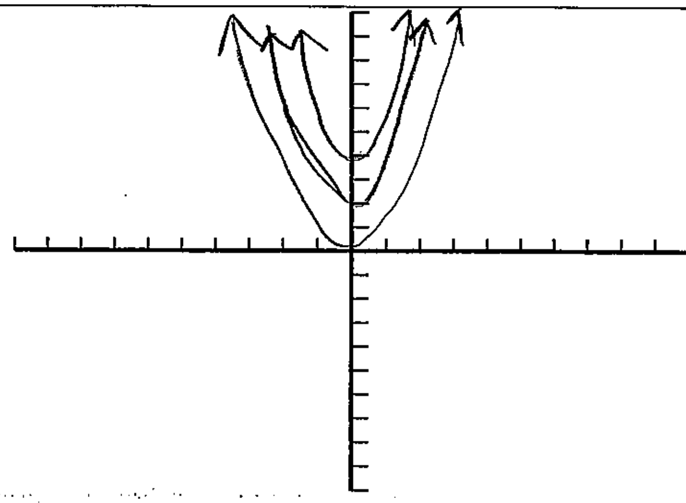
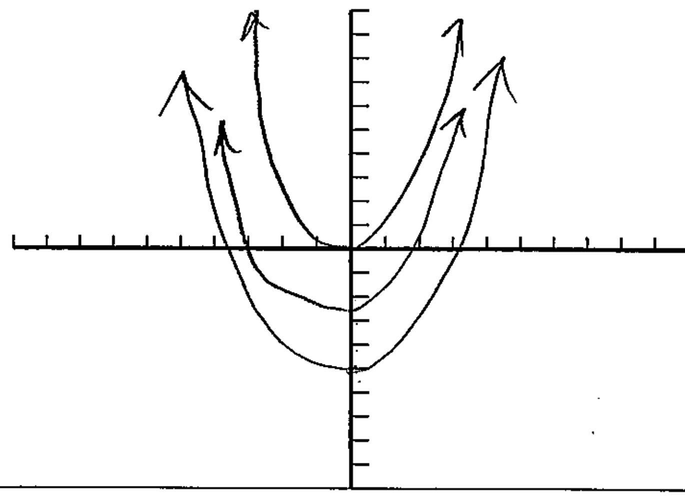
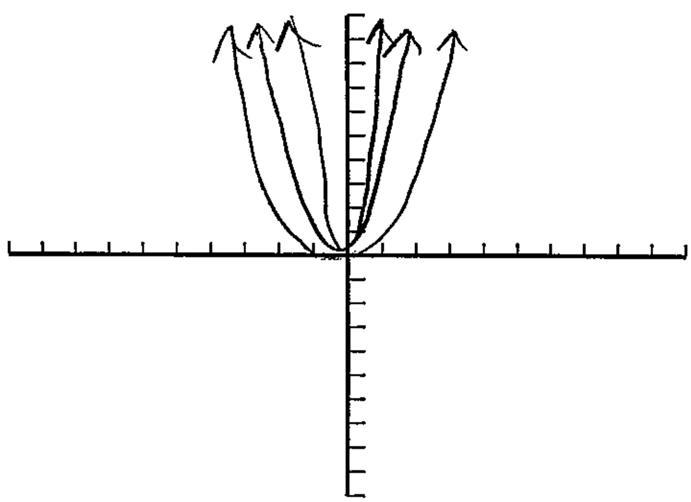
$f(x) = a(x-m)(x-n)$

Roots: $\{m, n\}$

11

Notes 11.3 - Graphing Quadratic Functions from Vertex Form

Graph the following equations using the graphing calculator. Sketch the graphs and answer the questions with each problem.

1.) • $f1(x)=x^2$ • $f2(x)=x^2+2$ • $f3(x)=x^2+4$ What happens to the graph when a number is added to x^2? What parameter will this affect? C	
2.) • $f1(x)=x^2$ • $f2(x)=x^2-5$ • $f3(x)=x^2-2.5$ What happens to the graph when a number is added to x^2? What parameter will this affect? C	
3.) • $f1(x)=x^2$ • $f2(x)=2x^2$ • $f3(x)=4x^2$ What happens to the graph when x^2 is multiplied by a number greater than 1? narrower What parameter will this affect? a	

4.)

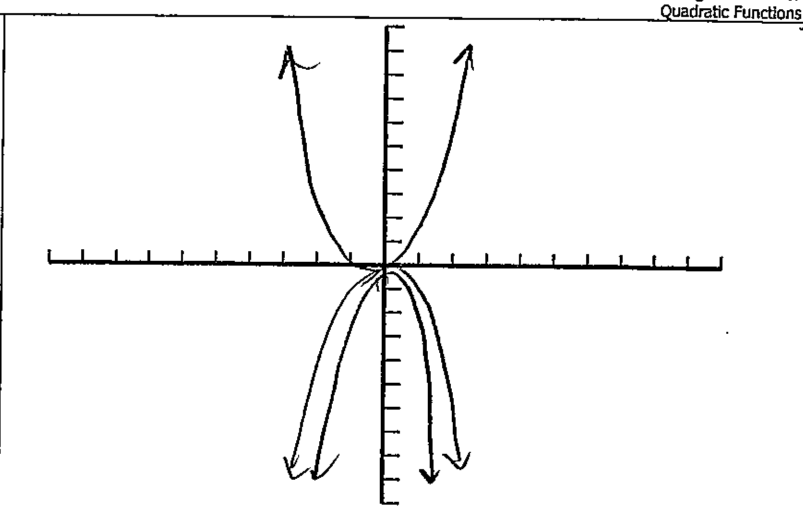
- $f1(x)=x^2$
- $f2(x)=-x^2$
- $f3(x)=-2x^2$

What happens to the graph when the coefficient of x^2 is negative?

reflects over x-axis

What parameter will this affect?

a

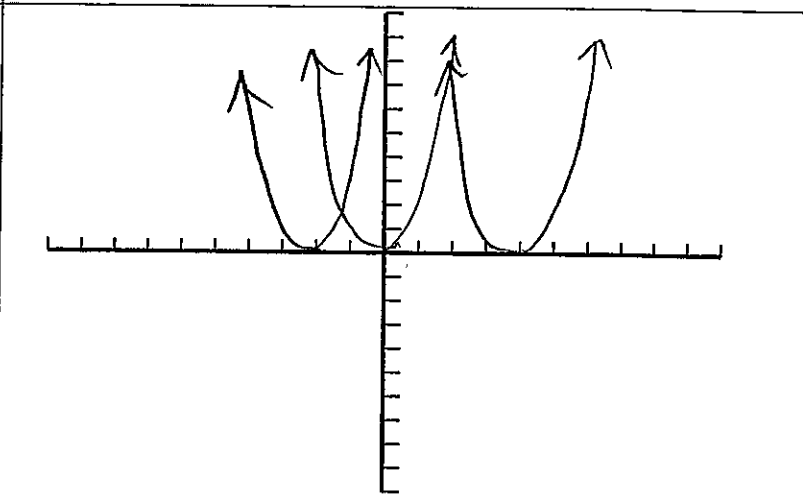


5.)

- $f1(x)=x^2$
- $f2(x)=(x-4)^2$
- $f3(x)=(x+2)^2$

What happens to the graph?

shifts left / right

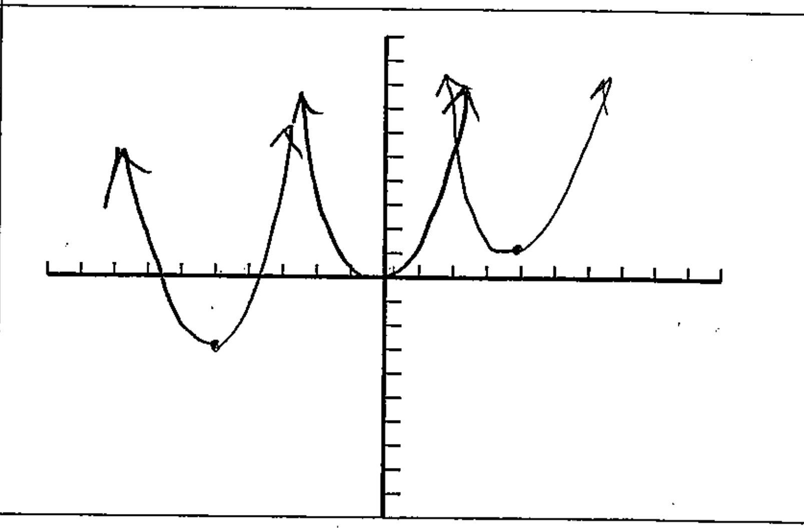


6.)

- $f1(x)=x^2$
- $f2(x)=(x+5)^2-2$
- $f3(x)=(x-4)^2+1$

What happens to the graph?

Shifts



13 For 7 – 10, use your graphing calculator to graph the functions (Type them exactly how you see them). Then, find the vertex of each graph. (Remember: Menu → 6: Analyze Graph → 2 or 3: Minimum or Maximum)

7.) $f(x) = (x-5)^2 + 4$

Vertex: (5,4)

8.) $g(x) = -2(x-3)^2 + 1$

Vertex: (-3,1)

9.) $h(x) = 3(x+4)^2 - 6$

Vertex: (-4,-6)

10.) $i(x) = x^2 + 5$

Vertex: (0,5)

11.) What relationship do you notice between the vertex and the equation of the quadratic function?

For 12 – 15, write a quadratic equation whose graph will have the given vertex:

12.) Vertex: (1.9, -4)

$f(x) = 3(x-1.9)^2 - 4$

13.) Vertex: (0,100)

$f(x) = -5x^2 + 100$

14.) Vertex: $\left(-2, \frac{3}{2}\right)$

$f(x) = 4(x+2)^2 + \frac{3}{2}$

15.) Vertex: (5,0)

$f(x) = -4(x-5)^2$

16. Find vertex of:

$f(x) = x^2 + 6x + 4$

$f(x) = x^2 + 6x + 9 + 4 - 9$

$f(x) = (x+3)^2 - 5$

vertex: (-3,-5)

min.

17.

$f(x) = 2x^2 + 10x + 6$

$f(x) = 2(x^2 - 5x + \frac{25}{4}) + 6 - \frac{25}{2}$

$\left(\frac{5}{2}\right)^2 = \left(\frac{25}{4}\right) =$

$f(x) = ($

$2(x - \frac{5}{2})^2 - \frac{13}{2}$

(2.5, -6.5)

Vertex Form of a Quadratic Function:

$f(x) = a(x-h)^2 + K$

where (h, K) is the vertex

Notes 11.4 - Graphing Quadratic Functions from Standard Form

1.) Find the equation of the axis of symmetry of a quadratic function that has roots $\{-7,9\}$.

$x = -\frac{7+9}{2} = \frac{2}{2} = 1$ $\boxed{x=1}$

2.) Find the equation of the axis of symmetry of a quadratic function with an equation $f(x) = x^2 - 6x + 8$.

$0 = (x-4)(x-2)$

$\{4, 2\}$

$\boxed{x=3}$

3.) Find a general equation to find the axis of symmetry for a quadratic function in standard form: $f(x) = ax^2 + bx + c$.

$$x = \frac{\left(\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}\right)}{2} = \frac{\left(-\frac{2b}{2a}\right)}{2} = \frac{\left(-\frac{b}{a}\right)}{2} = -\frac{b}{2a}$$

$\boxed{x = -\frac{b}{2a}}$

4.) Graph the equation $f(x) = x^2 - 6x + 5$ and identify the key features.

$x = \frac{6}{2} = 3$

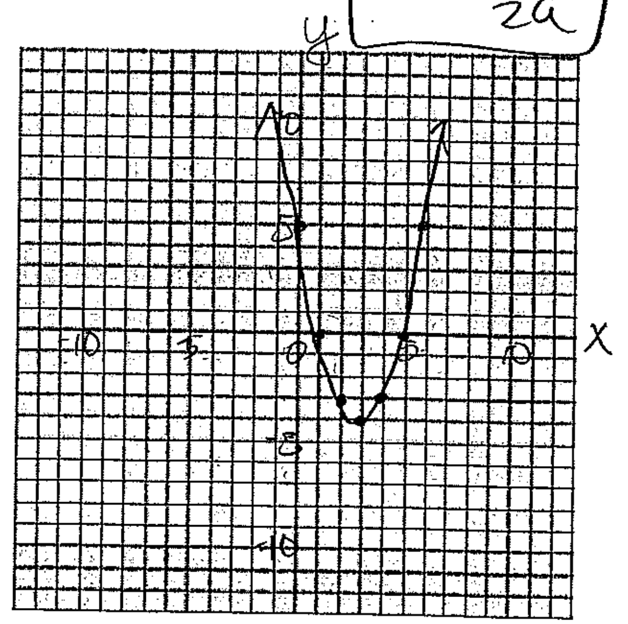
x	y
0	5
1	0
2	-3
3	-4
4	-3
5	0
6	5

Axis of sym:
$x = 3$

Vertex:
$(3, -4)$

y-int:
$(0, 5)$

Roots:
$\{1, 5\}$



5.) Consider the function $f(x) = 2x^2 - 8x - 10$.

a.) Find the vertex using the equation for the axis of symmetry.

$x = \frac{8}{2(2)} = \frac{8}{4} = 2$

$x = 2$

$f(2) = 2(2)^2 - 8(2) - 10$

$f(2) = 8 - 16 - 10$

$f(2) = -18$

$\boxed{(2, -18)}$

b.) Find the vertex by putting the function in vertex form.

$f(x) = 2(x^2 - 4x + 4) - 10 - 8$

$f(x) = 2(x - 2)^2 - 18$

$(2, -18)$

c.) Find the vertex by putting the function in factored form.

$f(x) = 2(x^2 - 4x - 5)$

$f(x) = 2(x - 5)(x + 1)$

$0 = 2(x - 5)(x + 1)$

$\{5, -1\}$

$x = \frac{5 - 1}{2} = \frac{4}{2} = 2$

$f(2) = -18$

$(2, -18)$

d.) What is the domain of the function?

~~domain~~ range

$\boxed{f(x) \geq -18}$

6.) Paige wants to start a summer lawn-mowing business. She comes up with the following profit function that relates the total profit to the rate she charges for a lawn-mowing job:

$$P(x) = -x^2 + 40x - 100$$

Both her profit and her rate are measured in dollars. Graph the function in order to answer the following questions.

Fix graph

$x = \frac{-40}{-2} = 20$

$P(20) = 300$ (20, 300)

x	y
0	-100
10	
20	
30	
40	

zeros: {2.68, 37.32}

a.) Graph $P(x)$.

b.) According to the function, what is her initial cost (e.g. maintaining the mower, buying gas, advertising)? Explain your answer in the context of this problem.

-$100.
Start-up fees are $100.

c.) Between what two prices does she have to charge to make a profit?

$2.68 and $37.32

d.) If she wants to make $275 profit this summer, is this the right business choice?

Yes- She can make up to $300.

Notes 11.5 - Applications of Quadratic Functions

1.) Chris stands on the edge of a building at a height of 60 feet and throws a ball upward with an initial velocity of 68 feet per second. The ball eventually falls all the way to the ground. The path of the ball can be modeled by the function $h(t) = -16t^2 + 68t + 60$.

a.) What units will we be using to solve this problem?

t - time in seconds
h(t) - height in feet

b.) What is the maximum point reached by the ball? After how many seconds will it take to reach that height?

$x = \frac{-68}{-32} = 2.125$

(2.125, 132.25)
max height is 132.25 ft at 2.125 sec.

c.) How long will it take the ball to land on the ground after being thrown? Show your work.

$0 = -16t^2 + 68t + 60$
$0 = -4(4t^2 - 17t - 15)$ → $0 = -4(4t+3)(t-5)$
$\{-3/4, 5\}$

5 sec

d.) How long will it take the ball to reach a height of 30 feet? Show your work. Round to the nearest one

$30 = -16t^2 + 68t + 60$
$0 = -16t^2 + 68t + 30$

$t = \frac{-68 \pm \sqrt{68^2 - 4(-16)(30)}}{-32}$

$\boxed{t = 4.7 \text{ sec}}$

e.) Find the average rate of change of the ball over the interval $1 \le x \le$ ~~3~~ 2. Explain what this represents. $f(0) = 60$ $f(2) = 132$

$\frac{132-60}{2-0} = \frac{72}{2} = 36$

The ball is traveling up at a rate of 36 ft/sec

f.) Compare your answer to part e to the average rate of change of the ball over the interval $3 \le x \le 5$. What might cause this change?

$f(3) = 120$ $f(5) = 0$

$\frac{0-120}{5-3} = \frac{-120}{2} = -60$

The ball is traveling down at a faster rate of 60 ft/s

2.) Find the quadratic equation that passes through the points (0,4), (1,9), and (–3,1).

$f(x)=ax^2+bx+c$
$f(0)=c$
$4=c$

$f(1)=a+b+c$
$9=a+b+c$

$f(-3)=9a-3b+c$
$1=9a-3b+c$

$9=a+b+4$
$1=9a-3b+4$

$a=5-b$
$1=9(5-b)-3b+4$
$1=45-9b-3b+4$
$1=49-12b$
$-48=-12b$
$4=b$

$a=1$

$\boxed{y=x^2+4x+4}$

3.) The table below represents the value of Andrew's stock portfolio, with *V* representing the value of the portfolio, in hundreds of dollars, and *t* is the time, in months, since he started investing. Answer the following question based on the table of values:

t (months)	*V*(*t*)
2	325
4	385
6	405
8	385
10	325
12	225
14	85
16	-95
18	-315

a.) What kind of function do you think this table represents? How do you know?

Quadratic - Follows a quadratic pattern

b.) Assuming this data is in fact quadratic, how much did Andrew invest in his stock initially? Explain how you arrived at this answer.

$225
pts of symmetry

c.) What is the maximum value of his stock and how long does it take to reach the maximum value?

$405 after 8 months

d.) If the pattern continues to follow the quadratic trend shown, do you advise Andrew to keep his stock portfolio? Explain why?

No, he'll keep losing money

e.) How fast is Andrew's stock value decreasing between [10,12]? Find another two-month interval where the average rate of change is faster than [10,12] and explain why.

$\frac{225-325}{12-10}=\frac{-100}{2}=\$-50/\text{mo.}$ $\boxed{[14,16]}$

f.) Find the equation of the function shown. $f(x)=-5x^2+60x+225$

Lab Notes - Discriminant

Look at the work shown for each of the following quadratic equations that are solved using the quadratic formula. Discuss with your partners how many solutions each equation has and why it may have that many solutions.

1.) $x^2 - 6x + 9 = 0$

$X = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$

$X = \frac{6 \pm \sqrt{36-36}}{2}$

$X = \frac{6 \pm \sqrt{0}}{2}$

$X = \frac{6}{2}$

$\boxed{\{3\}}$

2.) $x^2 - 6x + 8 = 0$

$X = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$

$X = \frac{6 \pm \sqrt{36-32}}{2}$

$X = \frac{6 \pm \sqrt{4}}{2}$

$X = \frac{6 \pm 2}{2}$

$X = \frac{6+2}{2}$ $\quad$ $X = \frac{6-2}{2}$

$X = 4$ $\quad$ $X = 2$

$\boxed{\{4, 2\}}$

3.) $x^2 - 2x - 7 = 0$

$X = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$

$X = \frac{2 \pm \sqrt{4+28}}{2}$

$X = \frac{2 \pm \sqrt{32}}{2}$

$X = \frac{2 \pm 4\sqrt{2}}{2}$

$\boxed{\{1+2\sqrt{2}, 1-2\sqrt{2}\}}$

4.) $x^2 + 4 = 0$

$X = \frac{0 \pm \sqrt{(0)^2 - 4(1)(4)}}{2(1)}$

$X = \frac{0 \pm \sqrt{-16}}{2}$

$\boxed{\varnothing}$

What is the discriminant?
$D = b^2 - 4ac$

Use your graphing calculator to determine how many zeros there are to the quadratic equations below. Also, determine the discriminant of each quadratic function.

5.) $f(x) = x^2 - 5x - 6$

Number of solutions: 2

Discriminant:

$(5)^2 - 4(1)(-6)$

$25 - 24 = 1$

6.) $f(x) = x^2 - 4x + 4$

Number of solutions: 0

Discriminant:

$(-4)^2 - 4(1)(4)$

$16 - 16 = 0$

7.) $f(x) = -2x^2 - 3x + 5$

Number of solutions: 2

Discriminant:

$(-3)^2 - 4(-2)(5)$

$9 + 40 = 49$

8.) $f(x) = 3x^2 + 2x + 3$

Number of solutions: 0

Discriminant:

$(+2)^2 - 4(3)(3)$

$4 - 36$

-32

What can you conclude?

If $D > 0$ – 2 real solutions

If $D = 0$ – 1 real solution

If $D < 0$ – 0 real solutions