Unit 12 Notes

Nature of and Transformations of Functions

Tentative Schedule

Day	Classwork	Assignment
Wed. 4/29	Finish Transformations of Graphs Activity	Video #12.1 – Vertical and Horizontal Shifts
Thurs. 4/30 Fri. 5/1	P.S. #12.1	Video #12.2 – Reflections over Axes and Vertical Stretches/Compressions
Mon. 5/4	P.S. #12.2	Video #12.3 – Horizontal Stretches/Compressions
Tues. 5/5 Wed. 5/6	P.S. #12.3	Video #12.4 – Systems of Equations (including non-linear equations)
Thurs. 5/7	P.S. #12.4	Finish problem set packet
Fri. 5/8 Mon. 5/11	Review for Test #12	Review for Test #12
Tues. 5/12	**Test #12**	REVIEW!

Name: _____

Transformations of Graphs

Shifts For c>0, *to obtain the graph of:*		
f(x)+c	*shift the graph of f(x)*	**upward** c units
f(x)-c	*shift the graph of f(x)*	**downward** c units
f(x+c)	*shift the graph of f(x)*	**left** c units
f(x-c)	*shift the graph of f(x)*	**right** c units

Stretches and compressions For c>1, *to obtain the graph of:*		
cf(x)	***stretch*** *the graph of f(x)*	**vertically** by a factor of c
(1/c)f(x)	***compress*** *the graph of f(x)*	**vertically** by a factor of c
f(cx)	***compress*** *the graph of f(x)*	**horizontally** by a factor of c
f(x/c)	***stretch*** *the graph of f(x)*	**horizontally** by a factor of c

Reflections *To obtain the graph of:*		
-f(x)	*reflect the graph of f(x)*	**about the x-axis**
f(-x)	*reflect the graph of f(x)*	**about the y-axis**

$f(x) = y$

Notes 12.1 - Vertical and Horizontal Shifts

Let's summarize what we have learned in the transformations of graphs activity.

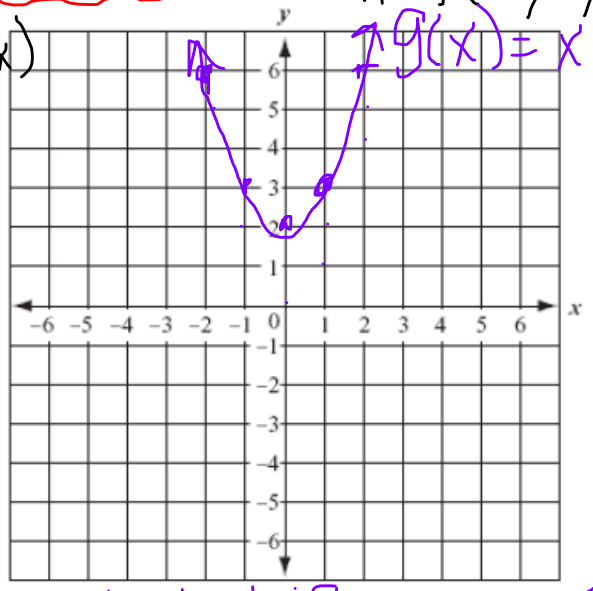
Vertical Shift	Shift **Up**	$f(x)+K$
	Shift **Down**	$f(x)-K$

Horizontal Shift	Shift **Right**	$f(x-h)$
	Shift **Left**	$f(x+h)$

Graph the following functions using their parent functions and your knowledge of vertical and horizontal shifts.

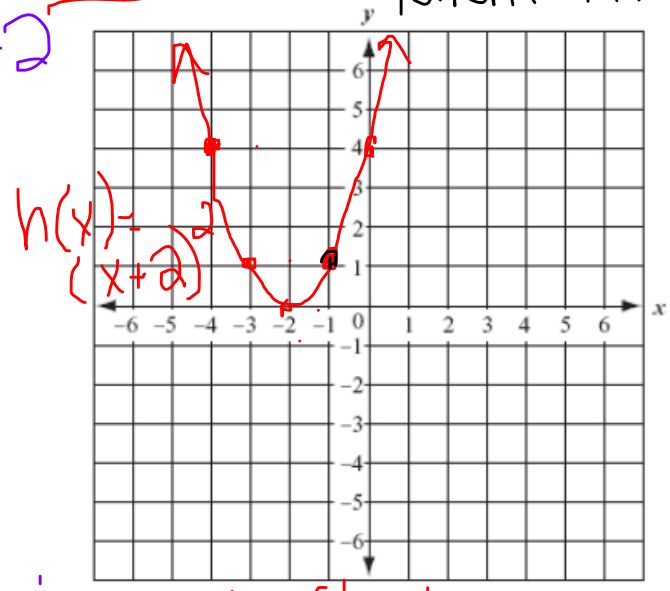
1.) $g(x) = x^2 + 2$ Parent: $f(x) = x^2$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4



vertical shift up of 2 units

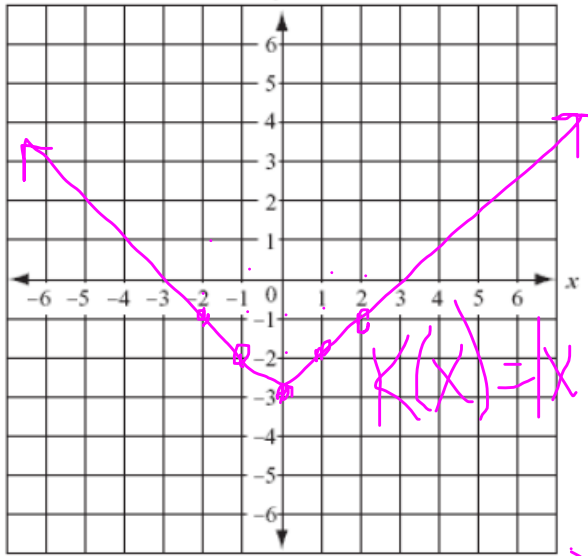
2.) $h(x) = (x+2)^2$ Parent: $f(x) = x^2$



shift left 2 units

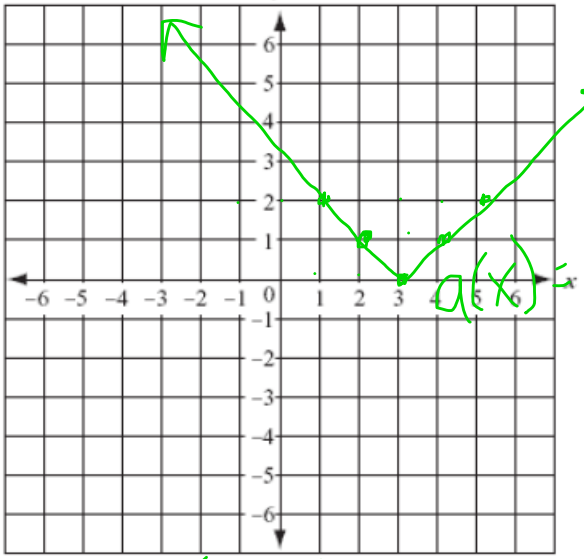
3.) $k(x) = |x| - 3$ Parent: $f(x) = |x|$

x	f(x)
-2	2
-1	1
0	0
1	1
2	2



shift down 3 units

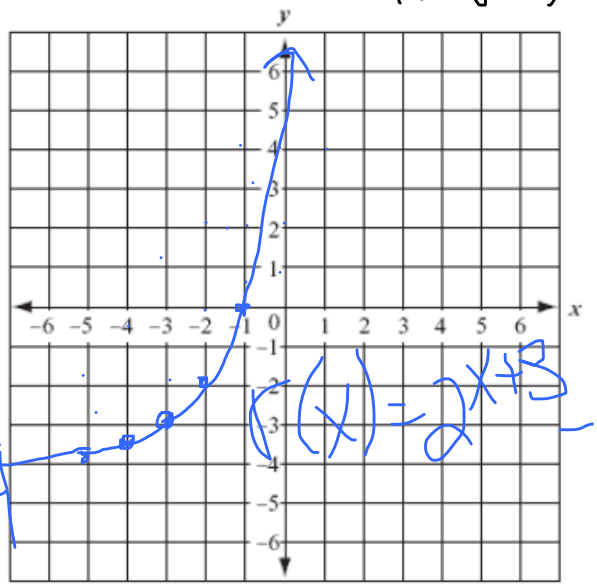
4.) $a(x) = |x-3|$ Parent: $f(x) = |x|$



shift right 3 units

5.) $r(x) = 2^{x+3} - 4$ Parent: $f(x) = 2^x$

x	$f(x)$
−2	¼
−1	½
0	1
1	2
2	4

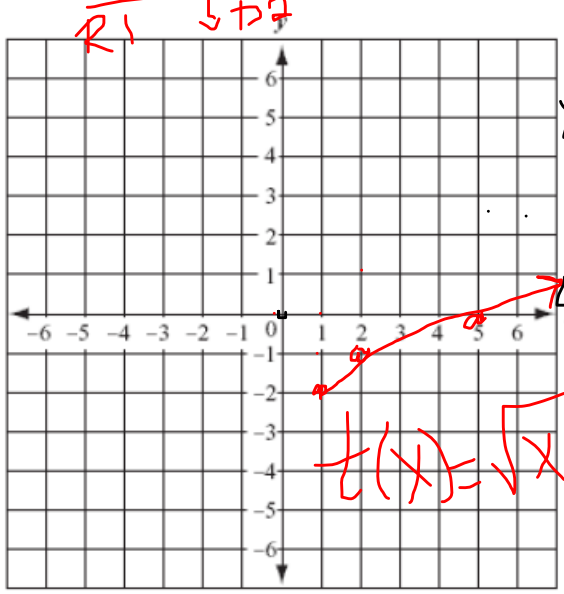


left 3
down 4
$r(x) = 2^{x+3} - 4$

6.) $t(x) = \sqrt{x-1} - 2$ Parent: $f(x) = \sqrt{x}$

R 1 ↓ by 2

x	$f(x)$
0	0
1	1
4	2



$t(x) = \sqrt{x-1} - 2$

7.) Given the parent function $f(x) = x^3$, find the equation of the function that is the transformation of $f(x)$ when shifted up 7 and right 8.

$f(x) = x^3 \rightarrow$ $g(x) = (x-8)^3 + 7$

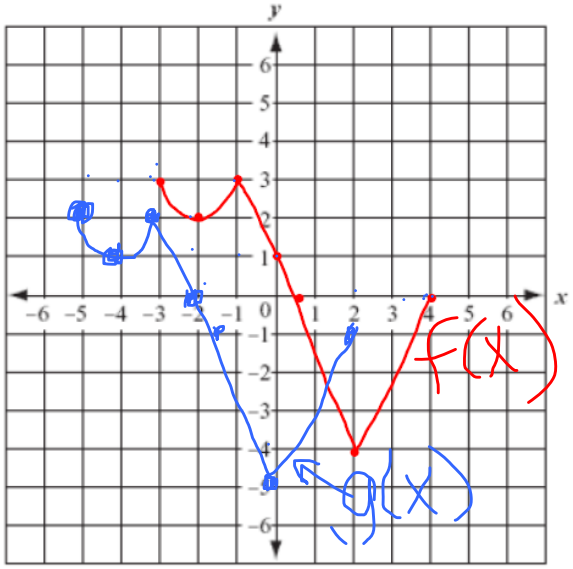
8.) Given the function $f(x) = 3(x-5)^2 + 4$, find the equation of the function that is the transformation of $f(x)$ when shifted left 6 and down 2.

$g(x) = 3(x-5+6)^2 + 4 - 2$

$g(x) = 3(x+1)^2 + 2$

(5, 4) → (−1, 2)

9.) The function shown below is $f(x)$. Draw in $g(x)$ if $g(x) = f(x+2) - 1$



- left 2
- down 1

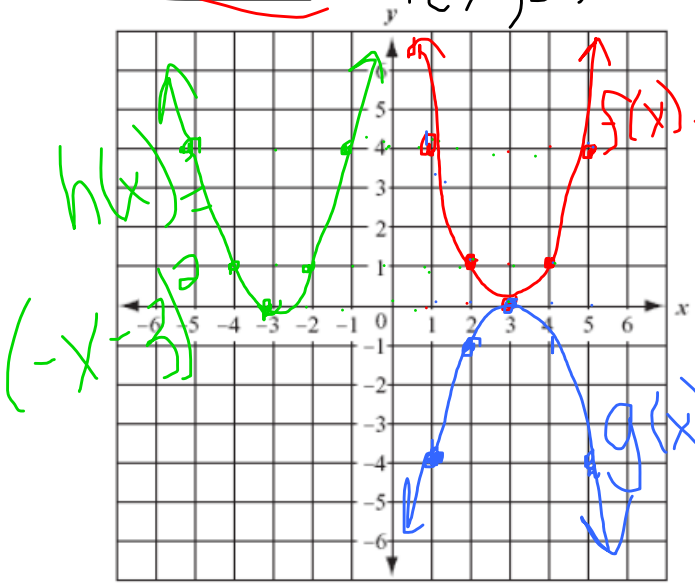
Notes 12.2 – Reflections Over Axes and Vertical Stretches and Compressions

Reminder:

Before Reflection (x, y)	→	After Reflection in the x-axis: $(x, -y)$
		After Reflection in the y-axis: $(-x, y)$

1.) Graph the following function below. Then, graph two reflections: one over the *x*-axis and one over the *y*-axis. Determine the equation of each function.

$f(x) = (x-3)^2$



Equation after a reflection over the *x*-axis:

$g(x) = -(x-3)^2$

Equation after a reflection over the *y*-axis:

$h(x) = (-x-3)^2$

Summary:

Before Reflection $f(x)$	→ (y values / x values)	After Reflection in the x-axis: $-f(x)$
		After Reflection in the y-axis: $f(-x)$

$f(x)$

Vertical Dilation	Vertical **Stretch**	$K \cdot f(x), K > 1$
	Vertical **Compression**	$K \cdot f(x), 0 < K < 1$

Horizontal Dilation	Horizontal **Stretch**	
	Horizontal **Compression**	

Graph the following functions using their parent functions and your knowledge of vertical stretches and compressions.

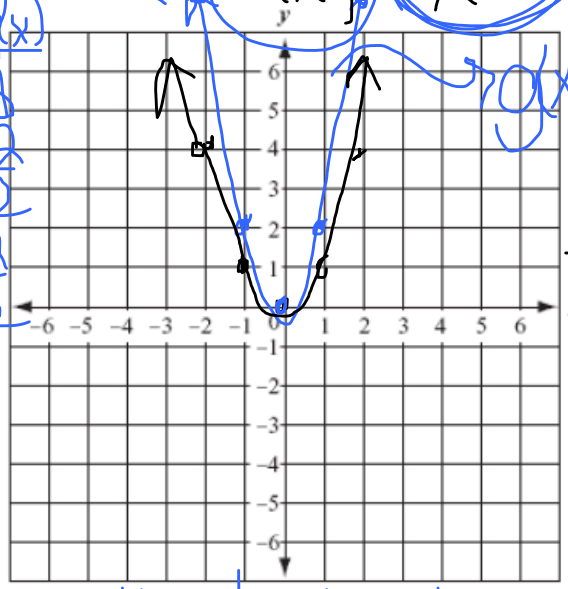
2.) $g(x) = 2x^2$

y is doubled

$f(x) = x^2$

$g(x) = 2x^2$

x	$f(x)$	$g(x)$
−2	4	8
−1	1	2
0	0	0
1	1	2
2	4	8



vertical stretch of s.f. 2

3.) $h(x) = \frac{1}{2}x^2$

$f(x) = x^2$

$f(x)$

$h(x)$

x	$f(x)$	$h(x)$
−2	4	2
−1	1	½
0	0	0
1	1	½
2	4	2

vertical compression of s.f. ½

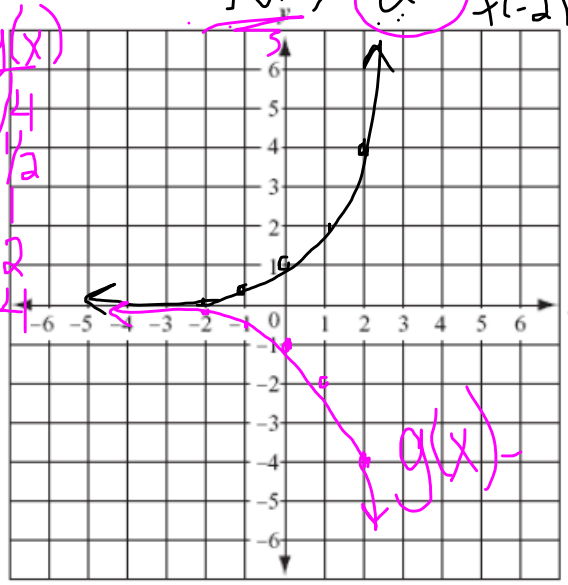
4.) $g(x) = -2^x$

$f(x) = 2^x$

$f(-2) = 2^{-2}$

$g(x)$

x	$f(x)$	$g(x)$
−2	¼	−¼
−1	½	−½
0	1	−1
1	2	−2
2	4	−4



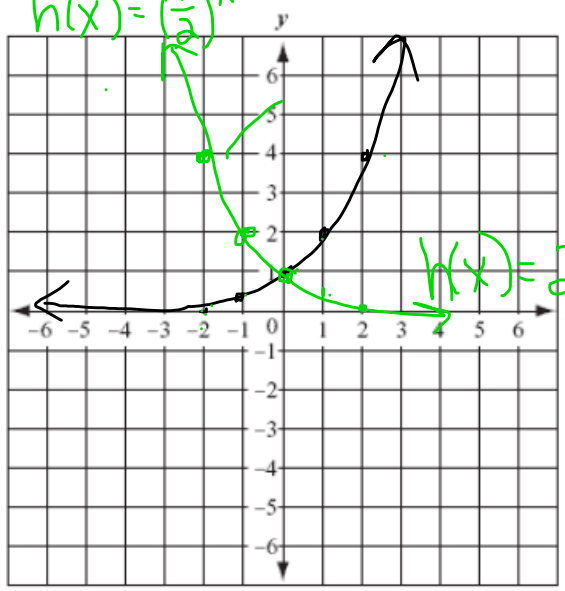
Reflection over the y-axis.

5.) $h(x) = 2^{-x}$

$h(x) = \left(\frac{1}{2}\right)^x$

$h(x) = 2^{-x}$

x	$f(x)$	$h(x)$
−2	¼	4
−1	½	2
0	1	1
1	2	½
2	4	¼



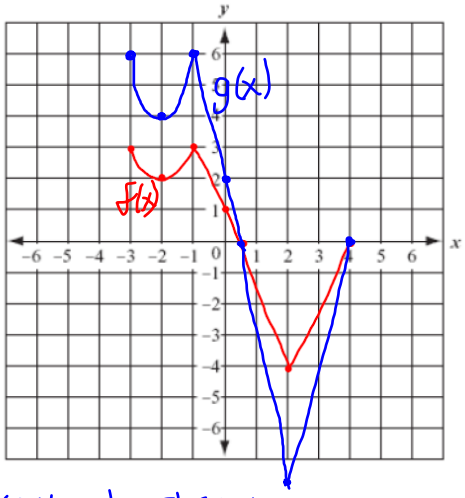
Reflection over the x-axis

Notes 12.3 - Horizontal Compressions

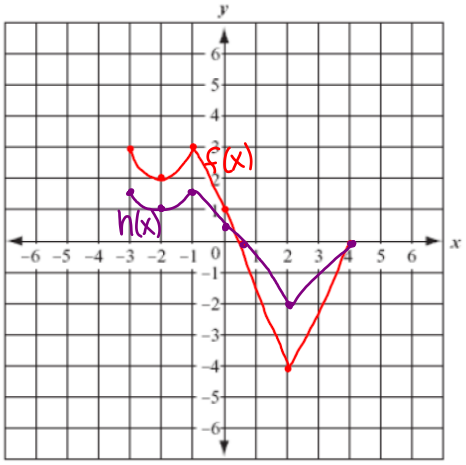
Vertical Dilation	Vertical **Stretch**	$k \cdot f(x), k > 1$	Horizontal Dilation	Horizontal **Stretch**	
	Vertical **Compression**	$k \cdot f(x)$, $0 < k < 1$		Horizontal **Compression**	

1.) Consider the graph of $f(x)$ below. Graph $g(x) = 2 \cdot f(x)$ and $h(x) = 0.5f(x)$. Describe each transformation.

x	f(x)	2·f(x)
-3	3	6
-2	2	4
-1	3	6
0	1	2
½	0	0
2	-4	-8
4	0	0



vertical stretch with a s.f of 2



vertical compression w/ a scale factor of $\frac{1}{2}$.

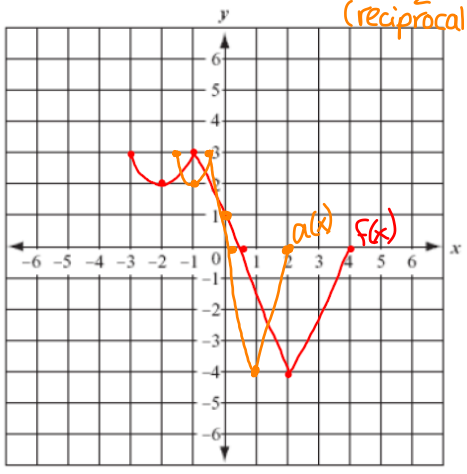
x	f(x)	0.5f(x)
-3	3	1.5
-2	2	1
-1	3	1.5
0	1	0.5
2	-4	-2
4	0	0

2.) Consider the graph of $f(x)$ below. Graph $a(x) = f(2x)$ and $b(x) = f(0.5x)$. Describe each transformation.

Multiply x by $\frac{1}{2}$

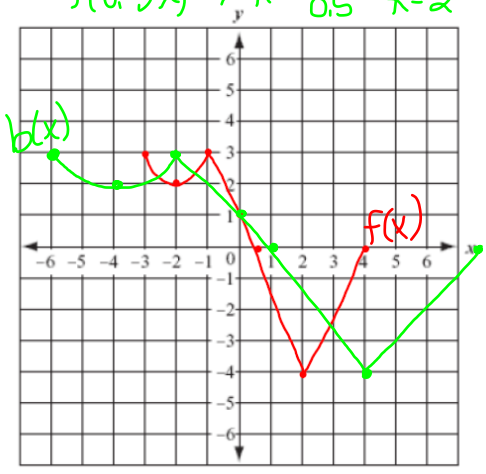
x		f(x)
-3	-1.5	3
-2	-1	2
-1	-0.5	3
0	0	1
0.5	0.25	0
2	1	-4
4	2	0

$f(2x) \rightarrow K = \frac{1}{2}$ (reciprocal)



Horizontal compression w/ a s.f. of $\frac{1}{2}$

$f(0.5x) \rightarrow K = \frac{1}{0.5}$ $K = 2$



Horizontal stretch of scale factor 2

multiply x by 2

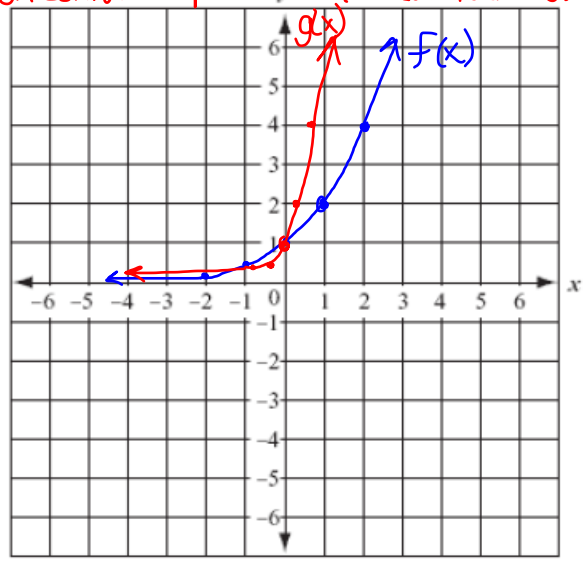
x		f(x)
-3	-6	3
-2	-4	2
-1	-2	3
0	0	1
0.5	1	0
2	4	-4
4	8	0

Graph the following functions using their parent functions and your knowledge of vertical and horizontal stretches and compressions.

3.) $g(x) = 2^{3x}$ Parent: $f(x) = 2^x$

horizontal compression w/ a scale factor of ⅓

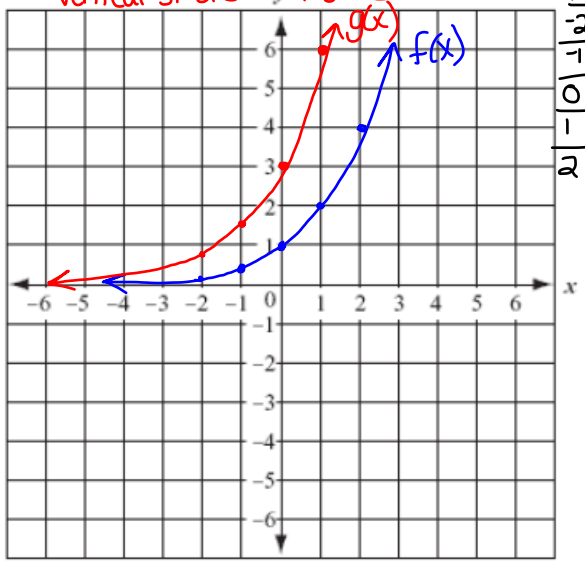
x	compress x by ⅓	f(x)
-2	-⅔	¼
-1	-⅓	½
0	0	1
1	⅓	2
2	⅔	4



4.) $g(x) = 3 \cdot 2^x$ parent function $f(x) = 2^x$

vertical stretch S.f. of 3

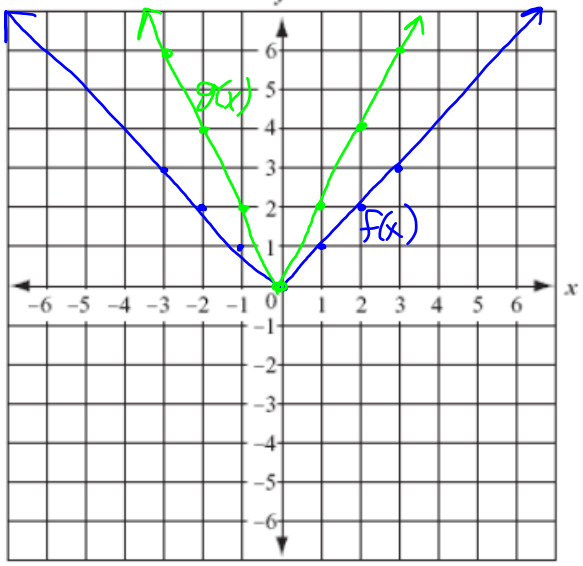
x	f(x)	3f(x)
-2	¼	¾
-1	½	3/2
0	1	3
1	2	6
2	4	12



5.) $g(x) = 2|x|$ Parent: $f(x) = |x|$

vertical stretch w/ a s.f. of 2.

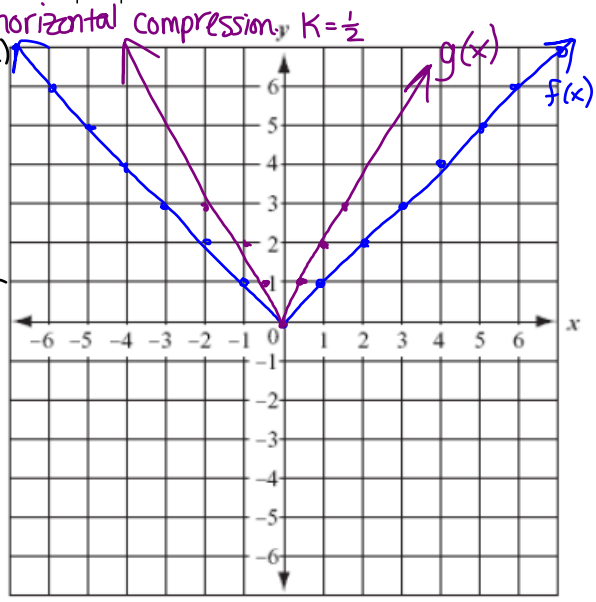
x	f(x)	2|x|
-3	3	6
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4
3	3	6



6.) $g(x) = |2x|$ Parent: $f(x) = |x|$

horizontal compression, $K = \frac{1}{2}$

x		f(x)
-3	-1.5	3
-2	-1	2
-1	-0.5	1
0	0	0
1	0.5	1
2	1	2
3	1.5	3



Notes 12.4 - Systems of Equations

1.) Solve the following system of equations graphically.

$y = x^2 - 6x + 3$ → Quad

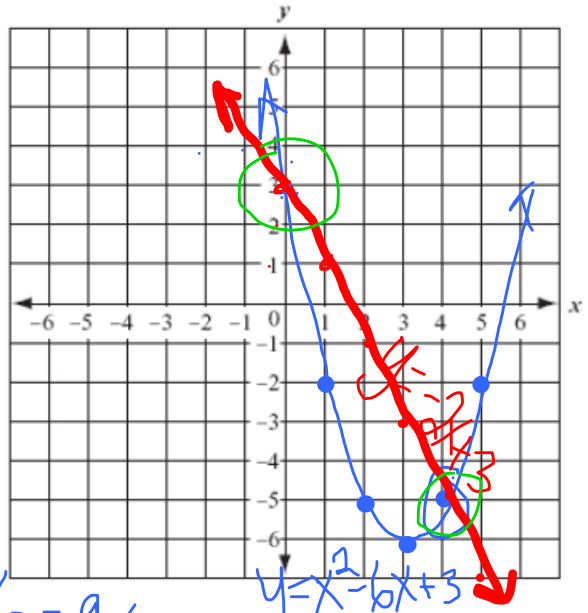
$y = -2x + 3$ → Linear

$y = x^2 - 6x + 3$ $\quad \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$y = x^2 - 6x + 9 + 3 - 9$

$y = (x-3)^2 - 6$

R3 ↓6



$y = (0-3)^2 - 6 = (-3)^2 - 6 = 9 - 6 = 3$

$\{(0,3) \text{ and } (4,-5)\}$

2.) Solve the following system of equations algebraically.

$y = x^2 - 6x + 3$ → Quad

$y = -2x + 3$ → Linear

$-2x + 3 = x^2 - 6x + 3$

$0 = x^2 - 4x$ ←

$0 = x(x-4)$

$x = 0$ | $x = 4$

$y = -2x + 3$ | $y = -2x + 3$

$y = -2(0) + 3$ | $y = -2(4) + 3$

$y = 0 + 3 = 3$ | $y = -8 + 3$

$y = -5$

$(0,3)$ $\quad (4,-5)$

3.) Solve the following system of equations algebraically.

$(x-2)^2+(y-1)^2=4$ →

$x+y=1$ ↑

$y=1-x$

$(x-2)^2+(y-1)^2=4$

$(x-2)^2+(1-x-1)^2=4$

$(x-2)^2+x^2=4$

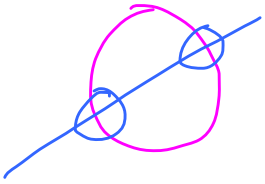
$x^2-4x+4+x^2=4$

$2x^2-4x+4=4$

$2x^2-4x=0$

$2x(x-2)=0$

$x=0 \mid x=2$



$x+y=1$

$0+y=1$

$y=1$

$x+y=1$

$2+y=1$

$y=-1$

$\{(0,1)+(2,-1)\}$