Unit 3 Notes

Algebraic Linear Equations
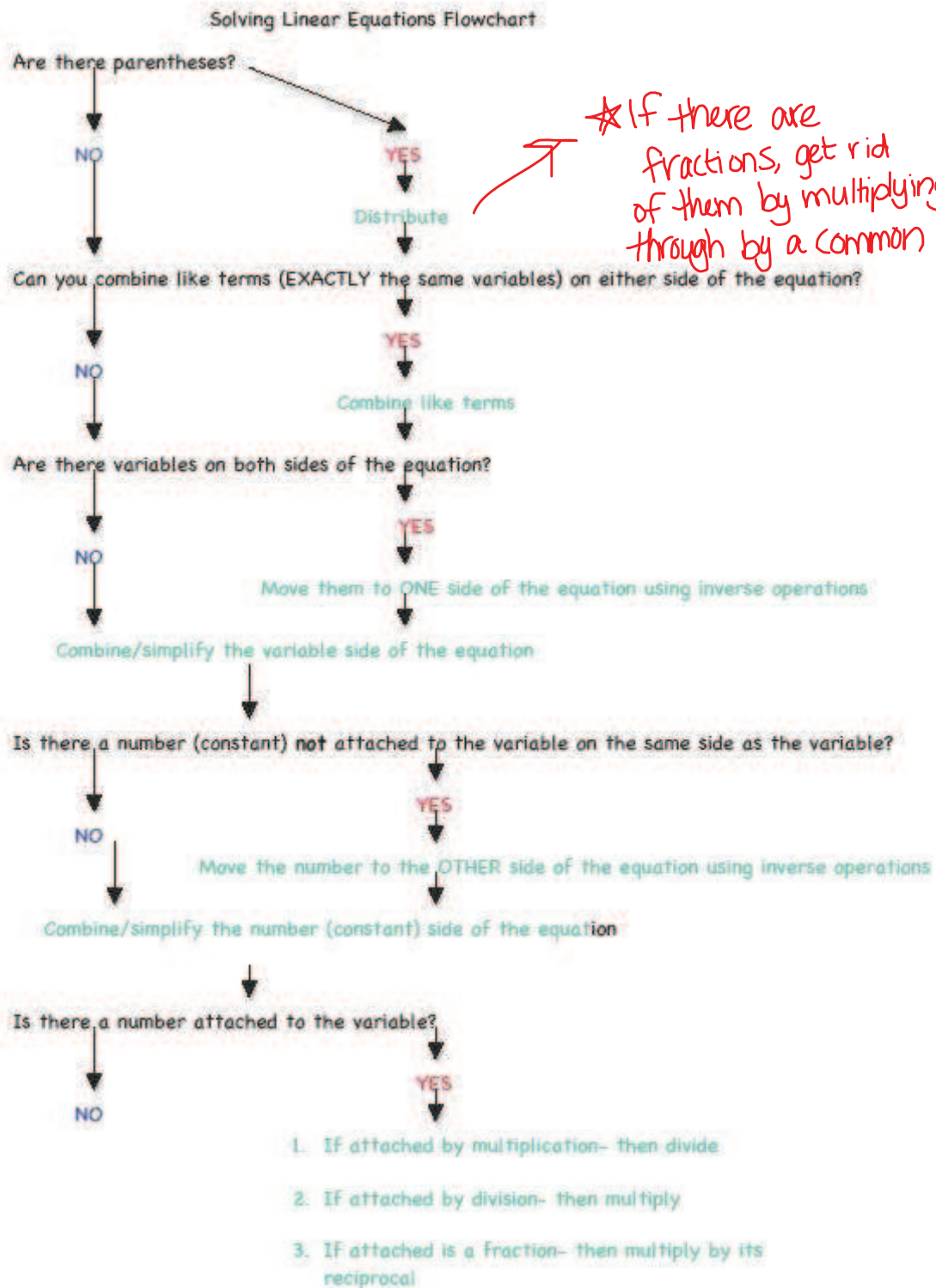
$$3x + 5 = 6(x + 4)$$

ditch the **3x**

$$3x + 5 = 6x + 24 \leftarrow \text{Be careful here too!}$$
$$\underline{-3x \qquad -3x}$$

$$5 = 3x + 24$$

ditch the **24**

$$\underline{-24 \qquad -24}$$

$$-19 = 3x$$

ditch the **3**

$$\frac{-19}{3} = \frac{3x}{3}$$

$$\frac{-19}{3} = x$$

Tentative Schedule

Day	Date	Topic	Assignment
1	Tues. 10/14 Fri. 10/15	Two-Step Equations P.S. #3.1	Video #3.2 with Notes: Equations that Involve Combining Like Terms and Distributing
2	Thurs. 10/16	P.S. #3.2	Video #3.3 with Notes Equations with Variables on Both Sides
3	Fri. 10/17 Mon. 10/20	P.S. #3.3	Video #3.4 with Notes Multi-Step Equations
4	Tues. 10/21	P.S. #3.4	Video #3.5 with Notes Applications of Word Problems
5	Wed. 10/22 Thurs. 10/23	**Quiz** P.S. #3.5	Finish all problem sets and corrections
6	Fri. 10/24	Special Cases of Equations	Watch Video #3.7 with Notes Isolating Variables
7	Mon. 10/27 Tues. 10/28	Finish up P.S. #3.6/3.7	Finish up P.S. #3.6/3.7
8	Wed. 10/29	Activity	Review for Test #3
9	Thurs. 10/30 Fri. 10/31	**Test #3**	Video #4.1 with Notes

Name: _____

Solving Equations Flowchart

Solving Linear Equations Flowchart

Are there parentheses?

NO   YES

Distribute

*If there are fractions, get rid of them by multiplying through by a common denominator.

Can you combine like terms (EXACTLY the same variables) on either side of the equation?

NO   YES

Combine like terms

Are there variables on both sides of the equation?

NO   YES

Move them to ONE side of the equation using inverse operations

Combine/simplify the variable side of the equation

Is there a number (constant) **not** attached to the variable on the same side as the variable?

NO   YES

Move the number to the OTHER side of the equation using inverse operations

Combine/simplify the number (constant) side of the equation

Is there a number attached to the variable?

NO   YES

1. If attached by multiplication– then divide

2. If attached by division– then multiply

3. If attached is a fraction– then multiply by its reciprocal

Notes 3.1 – Two Step Equations

When we want to <u>solve</u> <u>equations</u>, we always want to get x by _____

on one side.

So we do inverse operations:

addition: <u>subtraction</u>   subtraction: <u>addition</u>

multiplication: <u>division</u>   division: <u>multiplication</u>

Class examples:

1.) $4 - \dfrac{x}{5} = -1$

$-4 -4$

$-5 \cdot \dfrac{-x}{5} = -5 \cdot -5$

$\boxed{x = 25}$

2.) $4.7 + 0.25x = 6.2$

$-4.7 -4.7$

$0.25x = 1.5$

$\dfrac{0.25}{0.25} \dfrac{}{0.25}$

$\boxed{x = 6}$

3.) $-4 - a = 5$

$+4 +4$

$\dfrac{-a}{-1} = \dfrac{9}{-1}$

$\boxed{a = -9}$

Notes 3.2 – Solving Equations with Like Terms and Distributing

Like Terms: <u>terms with the same variable and exponent</u>

<u>Ex: $5x^2$ and $-8x^2$ Non-ex: $4x^3 + 2x^2$ non-ex: $5x, 4y$</u>

Combine the following like terms:

1.) $5 + 3y - 2x + 4x - 2y$

$\boxed{5 + y + 2x}$

Distribute:

2.) $5(w + 3)$

$\boxed{5w + 15}$

3.) $\dfrac{1}{3}(3x + 6)$ $\dfrac{1}{3} \cdot \dfrac{3}{1} = \dfrac{3}{3} = 1$

$1x + 2$ $\dfrac{1}{3} \cdot \dfrac{6}{1} = \dfrac{6}{3} = 2$

$\boxed{x + 2}$

4.) $-4(2x - 5)$

$-8x + 20$

Solve the following equations by first $\underline{\text{distributing}}$, then $\underline{\text{combine}}$
$\underline{\text{like}}$ $\underline{\text{terms}}$.

5.) $7(a-2)+5+2a=-18$

$7a-14+5+2a=-18$

$9a-9=-18$
$+9 \quad +9$

$\dfrac{9a}{9}=\dfrac{-9}{9}$

$\boxed{a=-1}$

6.) $4x-3(2x+8)=-12$

$4x-6x-24=-12$

$-2x-24=-12$
$+24 \quad +24$

$\dfrac{-2x}{-2}=\dfrac{12}{-2}$

$\boxed{X=-6}$

Check: PEMDAS
$4x-3(2x+8)=-12$
$4(-6)-3(2(-6)+8)=-12$
$4(-6)-3(-12+8)=-12$
$4(-6)-3(-4)=-12$
$-24+12=-12$
$-12=-12 \checkmark$

7.) $34=9-2x+5$

$34=14-2x$
$-14 \quad -14$

$\dfrac{20}{-2}=\dfrac{-2x}{-2}$

$\boxed{-10=X}$

Notes 3.3 - Equations with Variables on Both Sides

Solve the following equations:

1.) $2m - 2 = 6m - 4$

$\underline{-2m \qquad -2m}$

$-2 = 4m - 4$

$\underline{+4 \qquad\qquad +4}$

$\dfrac{2}{4} = \dfrac{4m}{4}$

$\dfrac{2}{4} = m$

$\boxed{\dfrac{1}{2} = m}$

2.) $2m - 2 = 6m - 4$

3.) $15 - \dfrac{1}{6}n = \dfrac{1}{6}n - 1$

$\qquad +\dfrac{1}{6}n \quad +\dfrac{1}{6}n$

$15 = \dfrac{2}{6}n - 1$

$\underline{+1 \qquad\qquad +1}$

$6 \cdot 16 = \dfrac{2}{6}n \cdot 6$

$\dfrac{96}{2} = \dfrac{2n}{2}$

$\boxed{48 = n}$

4.) $9w + 3 = 4w \boxed{- 9}$

$\underline{-4w \qquad -4w}$

$5w + 3 = -9$

$\underline{-3 \qquad\; -3}$

$\dfrac{5w}{5} = \dfrac{-12}{5}$

$\boxed{w = -12/5}$

Notes 3.4 - Multi-Step Equations

Steps to solving multi-step equations.

1.) _____. *Look @ the*

2.) _____ like terms. *flowchart on pg. 2 instead*

3.) Get the _____ on one side and the _____ on the other.

4.) _____

1.) $5(x + 3) + 3x = 2x - 12 + 3x$

$$5x + 15 + 3x = 2x - 12 + 3x$$

$$8x + 15 = 5x - 12$$
$$-5x \qquad -5x$$

$$3x + 15 = -12$$
$$-15 \quad -15$$

$$\frac{3x}{3} = \frac{-27}{3}$$

$$\boxed{x = -9}$$
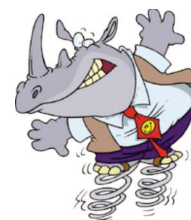
2.) $\frac{1}{2}\left(-4 + 6x\right) = \frac{1}{3}x + \frac{2}{3}\left(x + 9\right)$

$3 \cdot -2 + 3x^{\cdot 3} = \frac{1}{3}x^{\cdot 3} + \frac{2}{3}x^{\cdot 3} + 6 \cdot 3$ $\frac{2}{3} \cdot \frac{9}{1} = \frac{18}{3} = 6$

$$-6 + 9x = 1x + 2x + 18$$

$$-6 + 9x = 3x + 18$$
$$-3x \quad -3x$$

$$-6 + 6x = 18$$
$$+6 \qquad +6$$

$$\frac{6x}{6} = \frac{24}{6}$$

$$\boxed{x = 4}$$

3.) $10x + \dfrac{x \cdot \cancel{10}}{\cancel{10}} = 44 \cdot 10$

$10x + x = 440$

$\dfrac{11x}{11} = \dfrac{440}{11}$

$\boxed{x = 40}$

$11\overline{)440}$
$\quad \underline{-44}\downarrow$
$\qquad 00$ (quotient 40)

4.) $\dfrac{3x}{4} - \dfrac{2x+1}{4} = -1.5$

5.) ⑥ $\dfrac{2x}{3} \ominus \dfrac{2+x \cdot 6}{2} = -4 \cdot 6$

$2 \cdot (2x) - 3(2+x) = -24$

$4x - 6 - 3x = -24$

$x - 6 = -24$

$\underline{+6 \quad +6}$

$\boxed{x = -18}$

$\dfrac{2}{\cancel{6}} \cdot \dfrac{(2x)}{\cancel{3}}$

$3 \cancel{6} \cdot \dfrac{(2+x)}{\cancel{2}}$

Notes 2.2 - Applications of Linear Equations

Steps to Solving Word Problems
1.) Underline or highlight all given information.
2.) Determine what you want to find out.
3.) Draw a picture if you need to.
4.) Write let statements. *Ask yourself: How many things do I not know?*
5.) Write your equation.
6.) Solve it.
7.) Check it.
*Ask yourself: Does this answer make sense?

1.) Cassidy has a brother who is 3 years younger than she is. The sum of their ages is 23. How old are Cassidy and her brother?

Let Cassidy's age $= x = 13$

Let bro's age $= x - 3 = 13 - 3$
$$= 10$$

(+)

$$x + x - 3 = 23$$
$$2x - 3 = 23$$
$$2x = 26$$
$$x = 13$$

Cassidy is 13 yrs. old and her brother is 10 years old.

2.) Jared has pennies and dimes in his pocket. The number of pennies is three less than two times the number of dimes he has in his pocket. If he has a total of 46 cents, how many pennies and dimes does he have in his pocket?

3.) The greater of 2 numbers is 1 less than 3 times the smaller. If three times the greater number is 5 more than 8 times the smaller, find the numbers.

Let the smaller # = x = 8
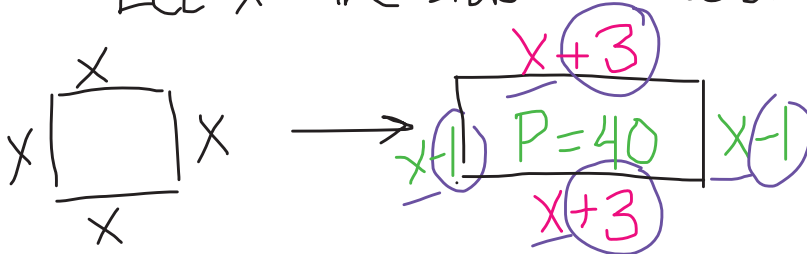Let the greater # = $3x-1$ = $3(8)-1$ = $24-1$ = 23

$\boxed{8,23}$

$$3(3x-1) = 8x +5$$
$$9x - 3 = 8x+5$$
$$x - 3 = 5 \longrightarrow x=8$$

4.) When two opposite sides of a square are increased by 3 cm, and the other two opposite sides are decreased by 1 cm, a rectangle with a perimeter of 40 results. Find the length of a side of the original square.

Let x = the side of the original square. = 9

$x+3$
$x-1$ P=40 $x-1$
$x+3$

$$4x + 4 = 40$$
$$4x = 36$$
$$x = 9$$

$\boxed{9 \text{ cm}}$

Consecutive Integers	
5	a
6	$a+1$
7	$a+2$

Consecutive Even Integers	
4	b
6	$b+2$
8	$b+4$

Consecutive Odd Integers	
5	c
7	$c+2$
9	$c+4$

5.) Find two consecutive integers whose sum is 45.

Let the 1st # = $\underline{x}$ = 22
Let the 2nd # = $\underline{x+1}$ = 23

$$x + x + 1 = 45$$
$$2x + 1 = 45$$
$$2x = 44$$
$$x = 22$$

$$\boxed{22 \, \xi \, 23}$$

6.) Three consecutive even integers are such that the sum of the smallest and 3 times the second exceeds twice the third by 38. Find the integers. $(+)$

Let the 1st even int = x = 20
Let the 2nd even int = $x+2$ = 22
Let the 3rd " " = $x+4$ = 24

$$x + 3(x+2) = 2(x+4) + 38$$
$$x + 3x + 6 = 2x + \underline{8 + 38}$$
$$4x + 6 = 2x + 46$$
$$\underline{-2x \qquad\quad -2x}$$
$$2x + 6 = 46$$
$$\underline{-6 \quad -6}$$
$$2x = 40 \rightarrow x = 20$$

Notes 3.6 - Number of Solutions of an Equation

Directions: For each equation, work with your partner to find the solution. If you get stuck on one, make an educated guess as to the solution and move on to the next so you are sure to get all three completed in the given time.

$2x + 4 = 2(x + 6)$	$4(x - 3) = 6x + 8$	$3(x - 2) = 3x - 6$
$2x+4=2x+12$ $-2x \quad -2x$ $\overline{\qquad 4=12 \ ???}$ No solution	$4x-12=6x+8$ $-4x \quad -4x$ $\overline{\qquad -12 = 2x+8}$ $-8 \qquad -8$ $\overline{\qquad -20 = 2x}$ $\dfrac{-20}{2} = \dfrac{2x}{2}$ $-10 = x$	$3x-6=3x-6$ $-3x \qquad -3x$ $\overline{\qquad -6=-6}$ ∞ Solutions

Try plugging in some values into each equation.

Nothing will work.	$4(x-3)=6x+8$ $4(-10-3)=6(-10)+8$ $4(-13)=6(-10)+8$ $-52=-60+8$ $-52=-52 \checkmark$	$3(2-2)=3(2)-6$ $3\cdot 0 = 6-6$ $0=0 \checkmark$ $3(5-2)=3(5)-6$ $3\cdot 3 = 15-6$ $9=9 \checkmark$ ★anything will work

Null Set: _No Solution_

(empty solution set)

Symbols: _$\emptyset$ or $\{\ \}$_

Identity: _Any real # is a solution_

	Null Set	One Solution	Identity
Words	No Solution	one solution	∞ Solutions
Algebra Representation	$ax + b = ax + c$	$ax + b = cx + d$ $a \neq c$	$ax + b = ax + b$
Example	$5x + 2 = 5x + 7$	$3x + 4 = 2x + 1$	$2x + 5 = 2x + 5$
Clues to look for	same coefficients, different constants	different coefficients	Identical sides (same coefficients and constants)

Notes 3.7 - Isolating Variables

Standard Equation	Literal Equation
Solve $2x + 3 = 9$ for x.	Solve $ax + b = c$ for x.

Standard Equation:
$$\begin{array}{c} 2x + 3 = 9 \\ \underline{-3 \quad -3} \\ \dfrac{2x}{2} = \dfrac{6}{2} \\ \boxed{x = 3} \end{array}$$

Literal Equation:
$$\begin{array}{c} ax + b = c \\ \underline{\quad -b \quad -b} \\ \dfrac{ax}{a} = \dfrac{c-b}{a} \end{array} \rightarrow \boxed{x = \dfrac{c-b}{a}}$$

For 1 – 8, isolate y.

1.) $y + 4 = 2x$
$$\underline{-4 \quad -4}$$
$$\boxed{y = 2x - 4}$$

2.) $y - 4 = 2x$
$$\underline{+4 \quad +4}$$
$$\boxed{y = 2x + 4}$$

3.) $2y = 10x + 8$
$$\dfrac{2y}{2} = \dfrac{10x}{2} + \dfrac{8}{2}$$
$$\boxed{y = 5x + 4}$$

4.) $x + 2y = 14$
$$\underline{-x \qquad -x}$$
$$\dfrac{2y}{2} = \dfrac{-1x}{2} + \dfrac{14}{2}$$
$$\boxed{y = -\dfrac{1}{2}x + 7}$$

5.) $x - y = 7$
$$\begin{array}{c} x - y = 7 \\ \underline{-x \qquad -x} \\ \dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{7}{-1} \\ \boxed{y = x - 7} \end{array}$$

$$\begin{array}{c} x - y = 7 \\ \underline{+y \quad +y} \\ x = y + 7 \\ \underline{-7 \quad -7} \\ \boxed{x - 7 = y} \end{array}$$

6.) $4x - 3y = 24$
$$\begin{array}{c} 4x - 3y = 24 \\ \underline{-4x \qquad -4x} \\ -3y = 24 - 4x \\ \underline{-3 \quad -3 \quad -3} \\ \boxed{y = -8 + \dfrac{4}{3}x} \end{array}$$

$$\begin{array}{c} 4x - 3y = 24 \\ \underline{+3y \quad +3y} \\ 4x = 3y + 24 \\ \underline{-24 \qquad -24} \\ \dfrac{4x - 24}{3} = \dfrac{3y}{3} \end{array} \rightarrow \boxed{\dfrac{4}{3}x - 8 = y}$$

7.) $x = 3(4 - y)$
$$\begin{array}{c} x = 12 - 3y \\ \underline{+3y \qquad +3y} \\ 3y + x = 12 \\ \underline{-x \quad -x} \\ \dfrac{3y}{3} = \dfrac{-x}{3} + \dfrac{12}{3} \\ \boxed{y = -\dfrac{1}{3}x + 4} \end{array}$$

8.) $7x - 2y = -22$
$$\begin{array}{c} 7x - 2y = -22 \\ \underline{-7x \qquad -7x} \\ \dfrac{-2y}{-2} = \dfrac{-7x}{-2} - \dfrac{22}{-2} \\ \boxed{y = \dfrac{7}{2}x + 11} \end{array}$$