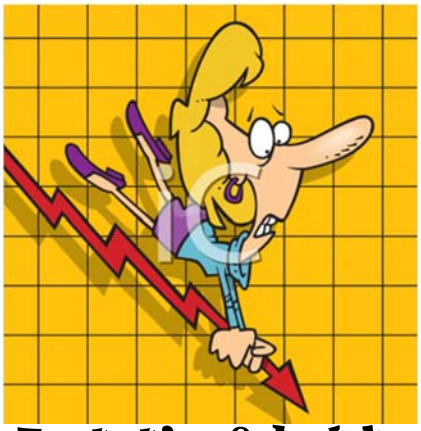
Unit 4 Notes

Lines and Linear Equations

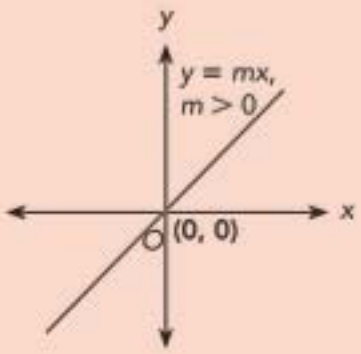


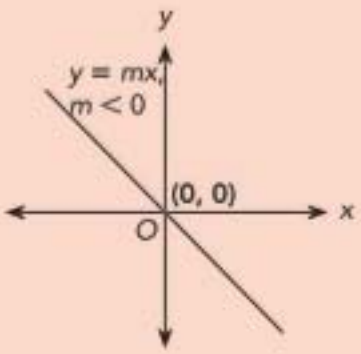
Tentative Schedule

Day	Date	Class Work	Assignment
	Mon. 11/3	Test #3	Watch Video #4.1 and Complete Notes Constant Rates
1	Tues. 11/4 Mon. 11/10	P.S. #4.1	Watch Video #4.2 and Complete Notes Understanding Slope
2	Wed. 11/12	P.S. #4.2	Watch Video #4.3 and Complete Notes Graphing Lines Using Points
3	Thurs. 11/13 Fri. 11/14	P.S. #4.3	Watch Video #4.4 and Complete Notes Graphing Lines Using Equations
4	Mon. 11/17	P.S. #4.4	Watch Video #4.5 and Complete Notes Finding Equations of Lines Day 1
5	Tues. 11/18 Wed. 11/19	P.S. #4.5 **Quiz #4.1**	Catch-up on Checklist
6	Thurs. 11/20	P.S. #4.5b	Watch Video #4.6 and Complete Notes Finding Equations of Lines Day 2
7	Fri. 11/21 Mon. 11/24	P.S. #4.6	Watch Video #4.7 and Complete Notes Applications of Linear Equations
8	Tues. 11/25	P.S. #4.7 **Quiz #4.2**	Catch-up on Checklist including P.S. #4.7
9	Mon. 12/1 Tues. 12/2	P.S. #4.8	Catch-up on Checklist
10	Wed. 12/3	Review for Test #4	Review for Test #4
11	Thurs. 12/4 Fri. 12/5	**Test #5**	Watch Video #5.1 with Notes

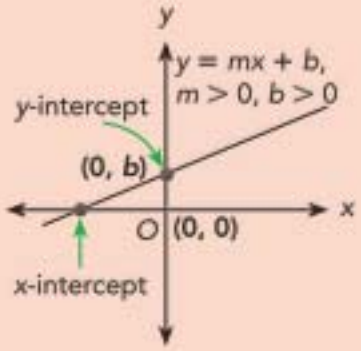
Name: _____

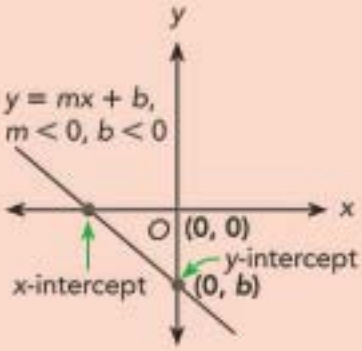
An equation of a line that passes through the origin, $O\,(0, 0)$ is $y = mx$.



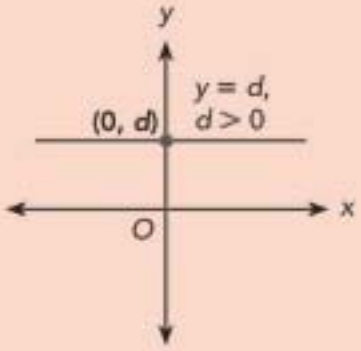


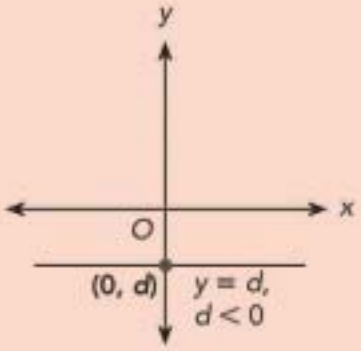
The equation of a line that intersects the y-axis at $(0, b)$ is $y = mx + b$.



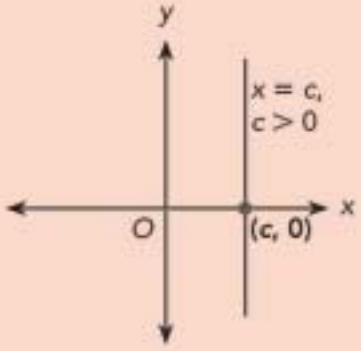


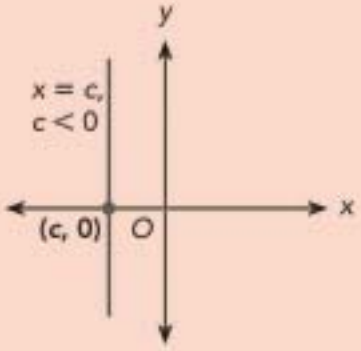
An equation of a straight line parallel to the x-axis and passing through the point $(0, d)$ is $y = d$, where d is the y-intercept.





An equation of a straight line parallel to the y-axis and passing through the point $(c, 0)$ is $x = c$, where c is the x-intercept.





Notes 4.1 – Understanding Rates

1.) If you leave home and walk in a given direction at a steady pace, your distance, *d* feet, from home is directly proportional to the time, *x* minutes, you walk. You can use a table and a graph to represent this proportional relationship.

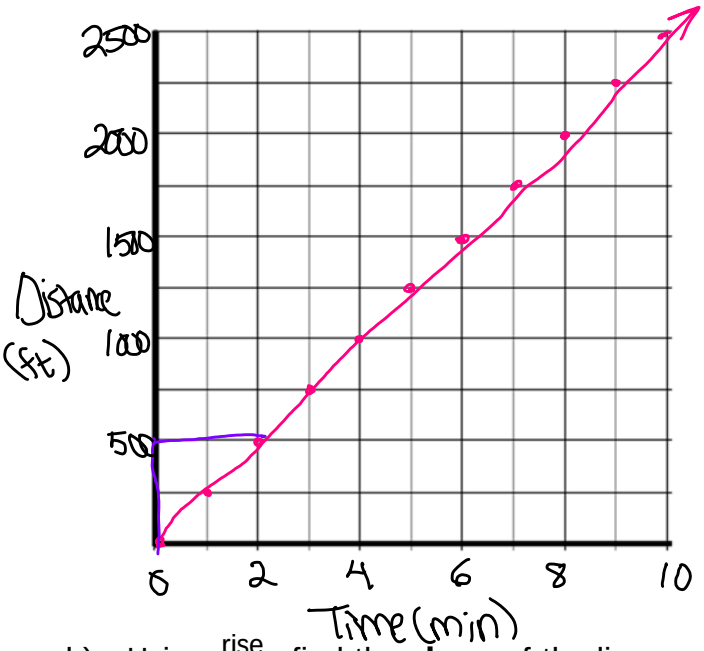
Time (*x* minutes)	1	2	3	4	5
Distance from Home (*d* feet)	250	500	750	1000	1250

a.) Fill in the rest of the chart.

b.) What is the constant of proportionality?

$k = \frac{\Delta y}{\Delta x} = \frac{250}{1} = 250$ ft/min.

c.) Graph the information given in the chart.



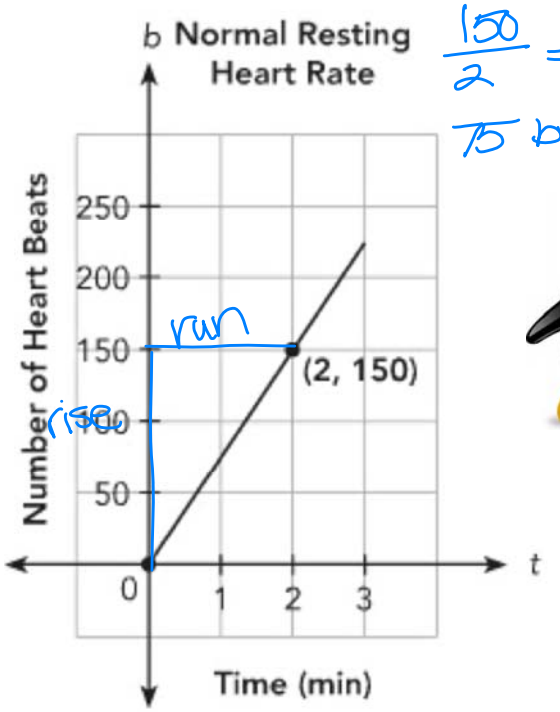
d.) Using $\frac{\text{rise}}{\text{run}}$, find the **slope** of the line.

$\frac{500}{2} = 250$ ft/min (same as constant of proportionality)

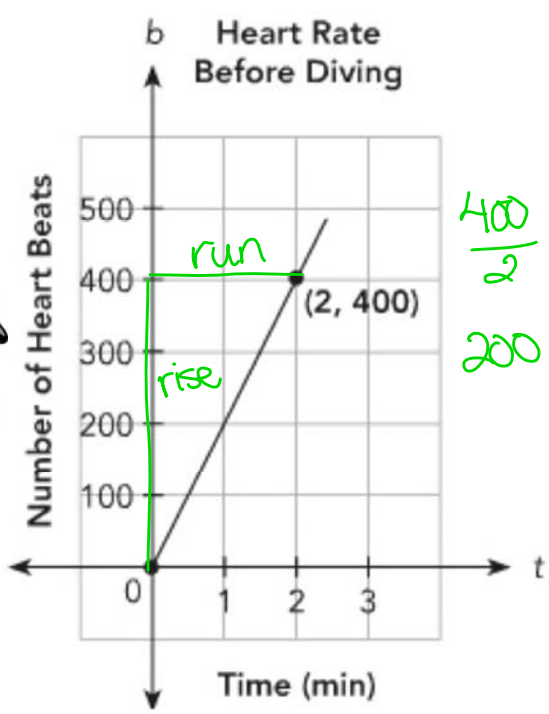
2.) Think of examples in which you would require rates of change.

- miles per hour
- hourly pay
- miles per gallon
- rent/mortgage
- tax

3.) The graphs give information about a penguin's number of heartbeats, *b*, over time, *t* minutes, during normal resting and just before diving. When is the penguin's heart rate greater, during normal resting, or just before diving?



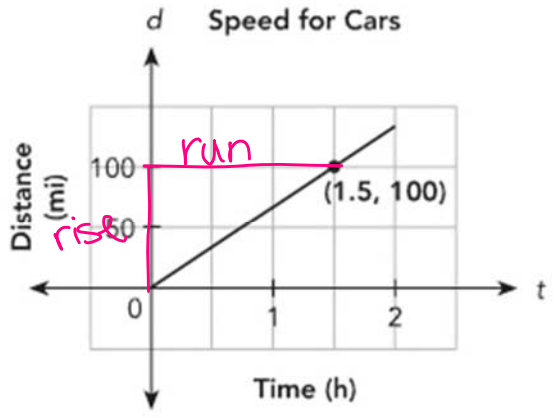
$\frac{150}{2} =$ 75 bpm



$\frac{400}{2} =$ 200 bpm

The heart rate is ~~faster~~ just before diving.

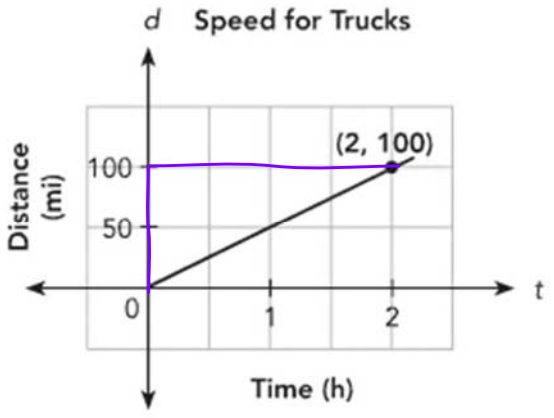
4.) The graphs give information about the distance, *d* miles, traveled over time, *t* hours, by cars and trucks on a California highway. Which speed is lower?



$\frac{\text{rise}}{\text{run}} = \frac{100}{1.5} = 100 \div \frac{3}{2} = 100 \cdot \frac{2}{3}$

$= \frac{200}{3} =$

$66\frac{2}{3}$ mph

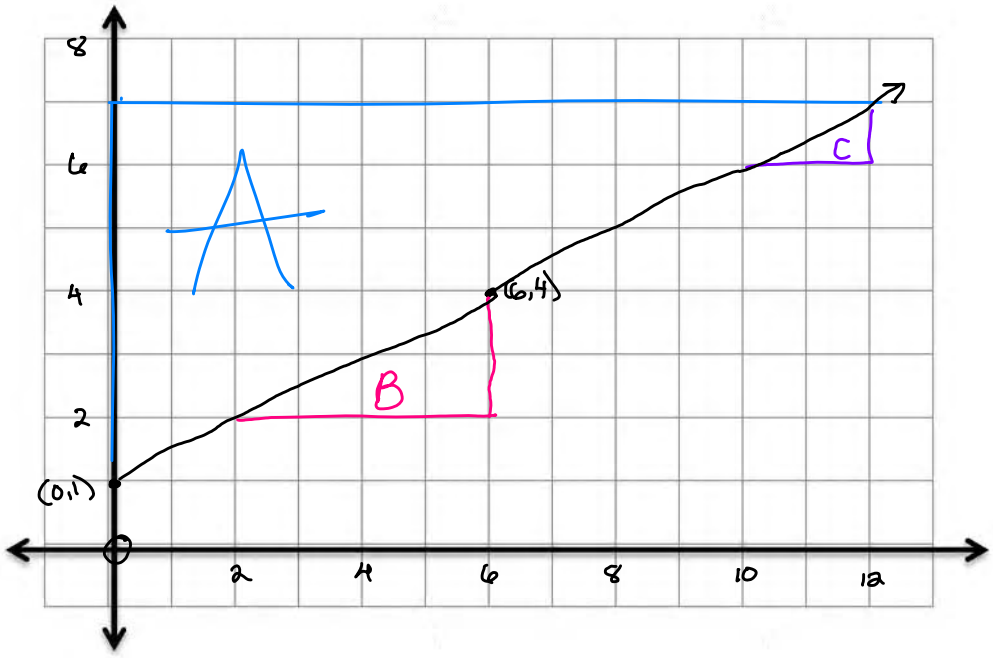


$\frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50$ mph

The truck is slower.

Notes 4.2 – Slope and Rate of Change

1.) Graph the line below on graph paper. The line should pass through the points (0,1) and (6,4). Then draw and label three right triangles on the line as shown. The triangles should be the same shape but different sizes. Make sure that each right angle lies on the intersection of two gridlines.



2.) Complete the table.

Triangle	Length of Vertical Side	Length of Horizontal Side	$\frac{\text{Length of vertical side}}{\text{Length of horizontal side}}$
A	6	12	$\frac{6}{12} = \frac{1}{2}$
B	2	4	$\frac{2}{4} = \frac{1}{2}$
C	1	2	$\frac{1}{2}$

3.) What do you notice about the last column of the table?

The ratio of the lengths of the vertical side to the horizontal side is always $\frac{1}{2}$.

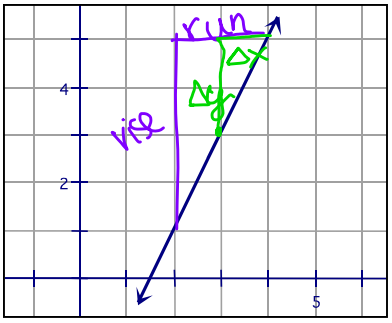
(It's constant.)

Slope Formula:

$m = \frac{\text{rise}}{\text{run}}$ $\quad m = \frac{\Delta y}{\Delta x}$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

Find the slope of each line.

1.)

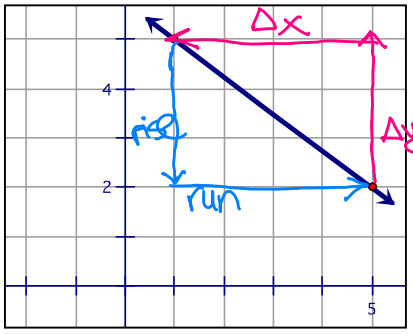


$m = \frac{rise}{run}$

$\frac{4}{2} = 2$

$m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$

2.)

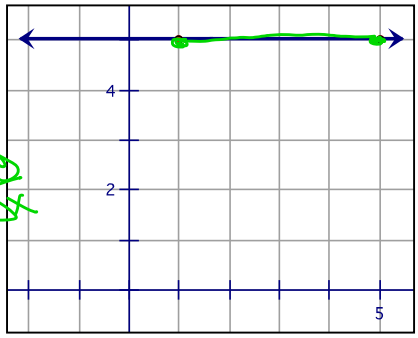


$m = \frac{rise}{run} = \frac{-3}{4}$

$m = \frac{\Delta y}{\Delta x} = \frac{3}{-4}$

$m = -\frac{3}{4}$

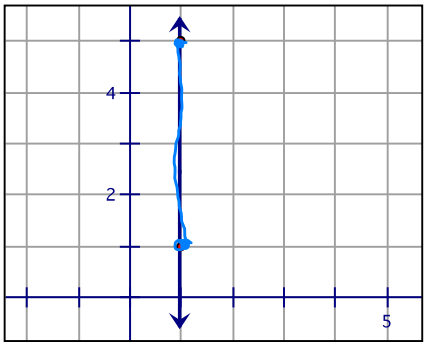
3.)



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{0}{4} = 0$ $\quad m = 0$

4.)



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{4}{0} =$

m = undefined

5.) When will the slope of a line be positive?

When Δy and Δx have the same sign

6.) When will the slope of a line be negative?

When Δy and Δx have opposite signs.

7.) Determine the slope of the line that passes through (2,-5) and (7,-10).

$x_1, y_1 \quad x_2, y_2$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - -5}{7 - 2} = \frac{-10 + 5}{7 - 2} = \frac{-5}{5} = -1$

$m = -1$

8.) Determine the slope of the line that passes through (-5,7) and (-5,9).

x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-7}{-5-(-5)} = \frac{2}{0}$ = undefined

(vertical line)

9.) Determine the slope of the line that passes through (-7,8) and (-9,8).

x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-8}{-9-(-7)} = \frac{0}{-2} = 0$

(horizontal line)

10.) Reece and Cassandra are riding the bus. There is a fee to use the bus and an additional rate per mile. Reece goes 5 miles and pays $7. Cassandra goes 9 miles and pays $10.20. How much is the rate per mile?

(x, y)

x: independent: distance (mi)

y: dependent: cost ($)

Reece: (5, 7) (x_1, y_1)

Cassandra: (9, 10.2) (x_2, y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10.2 - 7}{9 - 5} = \frac{3.2}{4} = 0.8$

↑ $3.20 for 4 miles

80¢/mile

(What's the fee? We'll discuss in the future.)

11.) Brianna and Kim join a gym. The gym has a joining fee and a monthly rate. If Brianna goes for 8 months and pays $262 and Kim goes for 11 months and pays $349, how much is the monthly rate?

(x, y)

x: independent: time (months)

y: dependent: cost ($)

Brianna: (8, 262)

Kim: (11, 349)

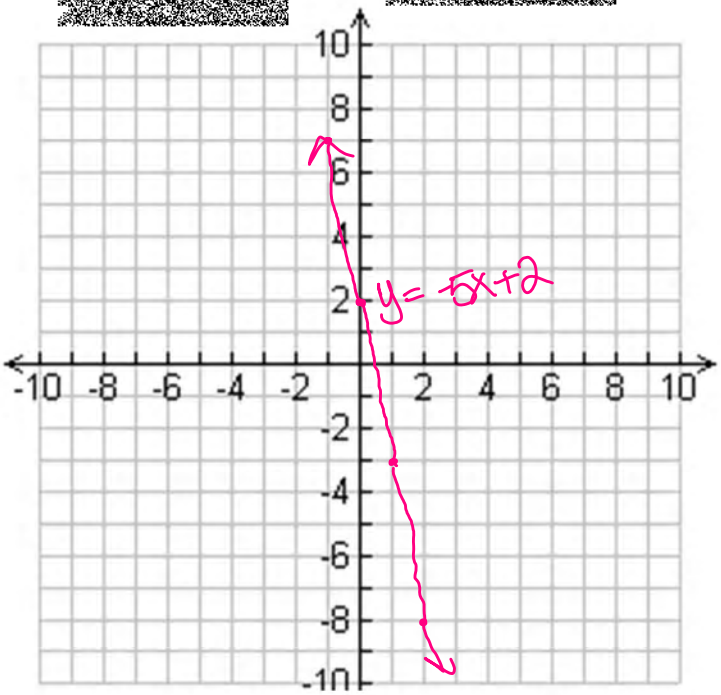
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{349 - 262}{11 - 8} = \frac{87}{3} = \$29/\text{mo}$

Notes 4.3 – Graphing Lines Using Charts

1.) $y = -5x + 2$

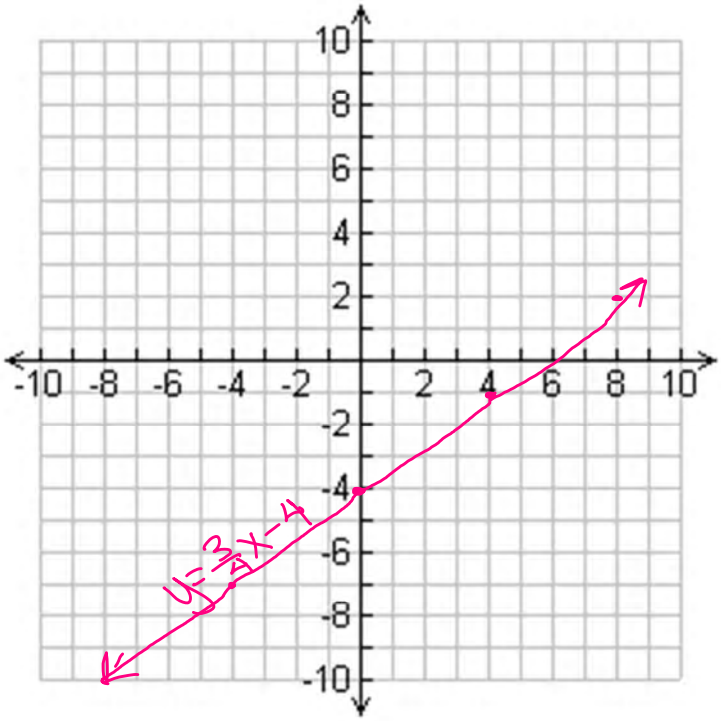
Replace x.

x	$y = -5x + 2$	y	Coordinate
-1	$-5(-1)+2 = 5+2$	7	(-1, 7)
0	$-5(0)+2 = 0+2$	2	(0, 2)
1	$-5(1)+2 = -5+2$	-3	(1, -3)
2	$-5(2)+2 = -10+2$	-8	(2, -8)



3.) $3x - 4y = 16$ ~~Isolate~~ y first.

x	$y = \frac{3}{4}x - 4$	y	Coordinate
-8	$\frac{3}{4}(-8) - 4 = -6 - 4$	-10	(-8, -10)
-4	$\frac{3}{4}(-4) - 4 = -3 - 4$	-7	(-4, -7)
0	$\frac{3}{4}(0) - 4 = 0 - 4$	-4	(0, -4)
4	$\frac{3}{4}(4) - 4 = 3 - 4$	-1	(4, -1)
8	$\frac{3}{4}(8) - 4 = 6 - 4$	2	(8, 2)



$3x - 4y = 16$

$+4y \quad +4y$

$3x = 4y + 16$

$-16 \quad -16$

$\frac{3x - 16}{4} = \frac{4y}{4}$

$y = \frac{3}{4}x - 4$

Notes 4.4 – Graphing Lines Using Equations

Look at the equations of the lines that are shown to you.

Write down some observations.

★ Graphs intersect at y-axis at the same number as the constant in the equation

★ Slope is the coefficient of x.

Slope-Intercept Form

$y = mx + b$

m: slope

b: y-intercept

Graph the following lines on the set of axes below.

1.) $3y = 12x - 21$

$\frac{3}{3} \quad \frac{3}{3} \quad \frac{3}{3}$

Equation in slope-intercept form:

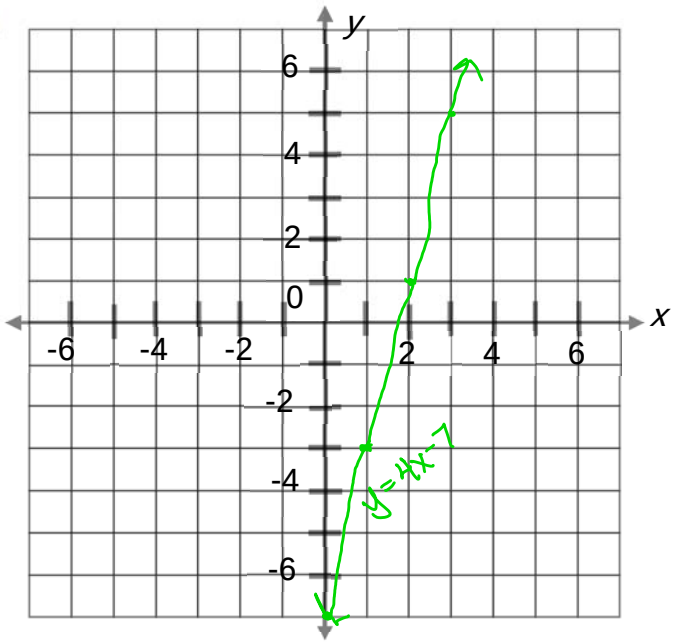
$y = 4x - 7$

Slope:

$\frac{4}{1}$ → rise, → run (up 4, right 1)

Y-intercept:

-7



2.) $-2x - 8y = -24$

$+8y \quad +8y$

$-2x = 8y - 24$

$+24 \quad +24$

$-2x + 24 = 8y$

$\frac{8y}{8} = \frac{-2x}{8} + \frac{24}{8}$

$y = -\frac{1}{4}x + 3$

Equation in slope-intercept form:

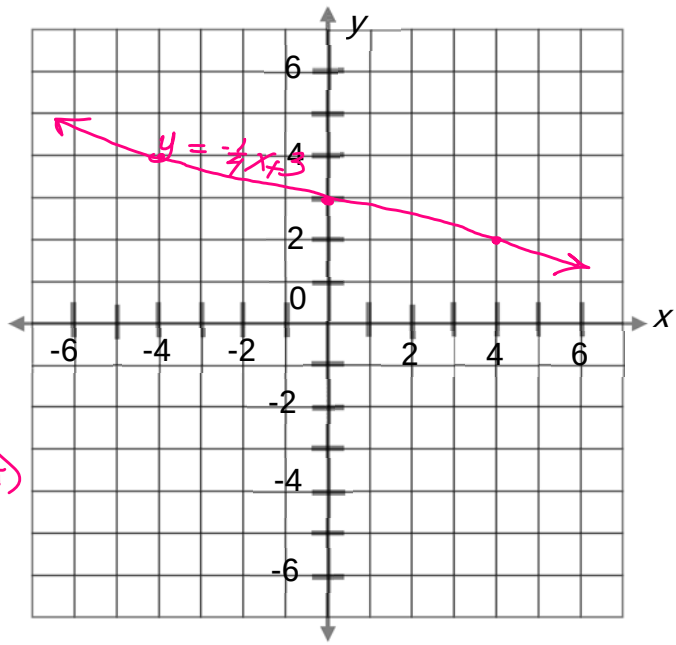
$y = -\frac{1}{4}x + 3$

Slope:

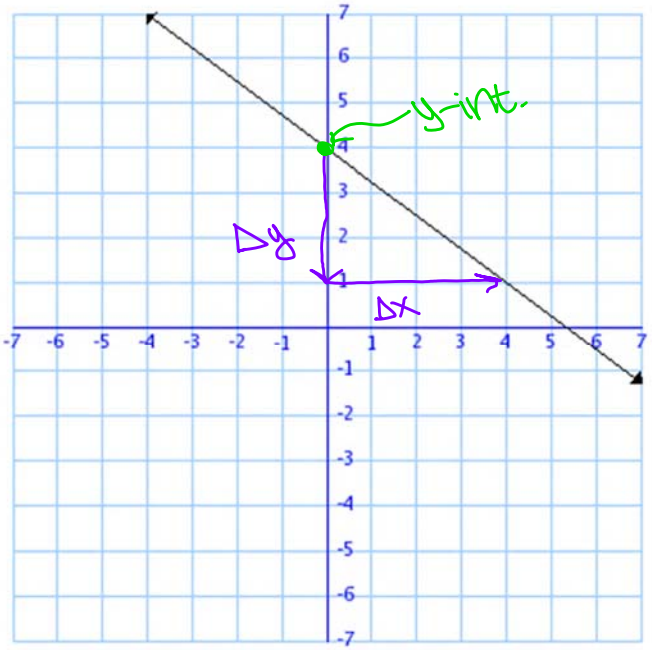
$-\frac{1}{4}$ $\frac{\Delta y}{\Delta x}$ (down 1, right 4 or up 1, left 4)

Y-intercept:

3



Determine the equations of the lines shown in the following graphs.

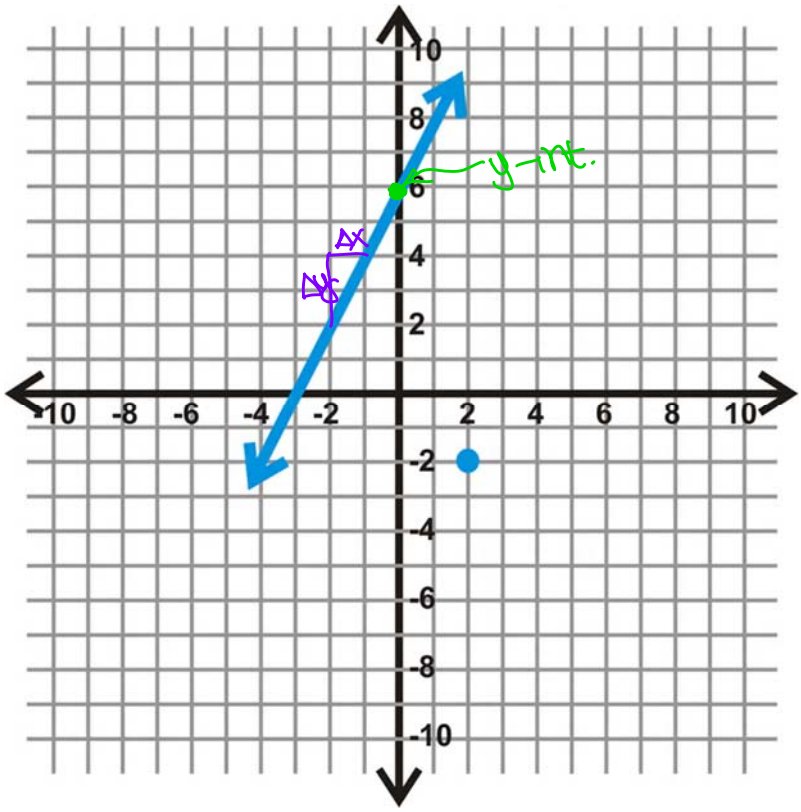


$m = \frac{\Delta y}{\Delta x} = \frac{-3}{4}$

$b = 4$

$y = mx + b$

$y = -\frac{3}{4}x + 4$



$m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$

$b = 6$

$y = mx + b$

$y = 2x + 6$

Notes 4.5 – Finding Equations of Lines Day 1

Slope-Intercept Form	Point-Slope Form
$y=mx+b$ m: slope b: y-intercept	$y-y_1=m(x-x_1)$ m: slope (x_1, y_1): point

1.) Find the equation of a line that has a slope of ¼ and a y-intercept of 3.

$m=\frac{1}{4}$

$b=3$

$y=mx+b$

$\boxed{y=\frac{1}{4}x+3}$

2.) Find the equation of a line that has a slope of -2 and a y-intercept of 2.

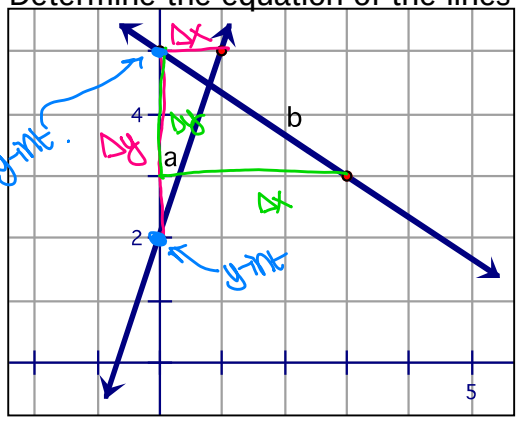
$m=-2$

$b=2$

$y=mx+b$

$\boxed{y=-2x+2}$

3.) Determine the equation of the lines shown in the following graphs.



a.) $m=\frac{\Delta y}{\Delta x}=\frac{3}{1}=3$ $b=2$

$\boxed{y=3x+2}$

b.) $m=\frac{\Delta y}{\Delta x}=\frac{-2}{3}$ $b=5$

$\boxed{y=-\frac{2}{3}x+5}$

4.) Find the equation of a line that has a slope of 2 and passes through the point (3,1) using slope-intercept form.

x y

Plug these values in.

$m=2$

$b=?$

$y=mx+b$

$y=2x+b$

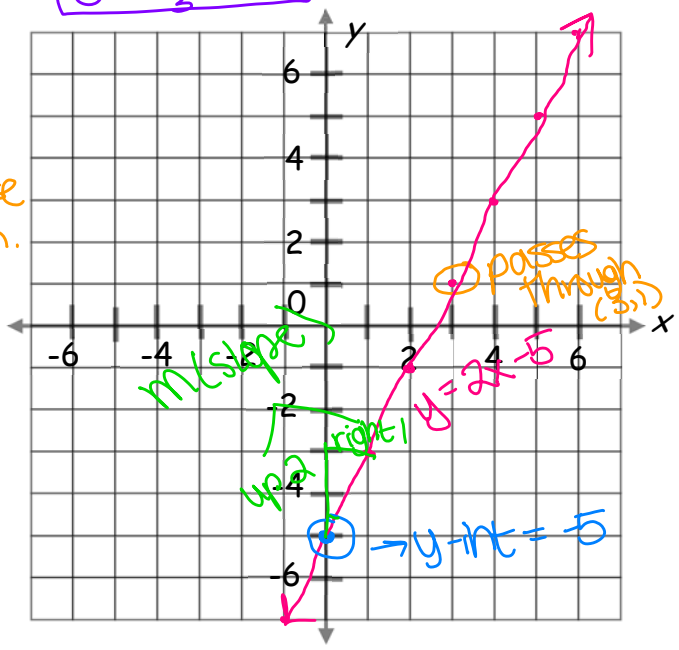
$1=2(3)+b$

$1=6+b$

$-6\quad -6$

$-5=b$

$\boxed{y=2x-5}$



5.) Find the equation of a line that has a slope of 2 and passes through the point (3,1) using point-slope form.

$y - y_1 = m(x - x_1)$ (m, x_1, y_1)

$y - 1 = 2(x - 3)$

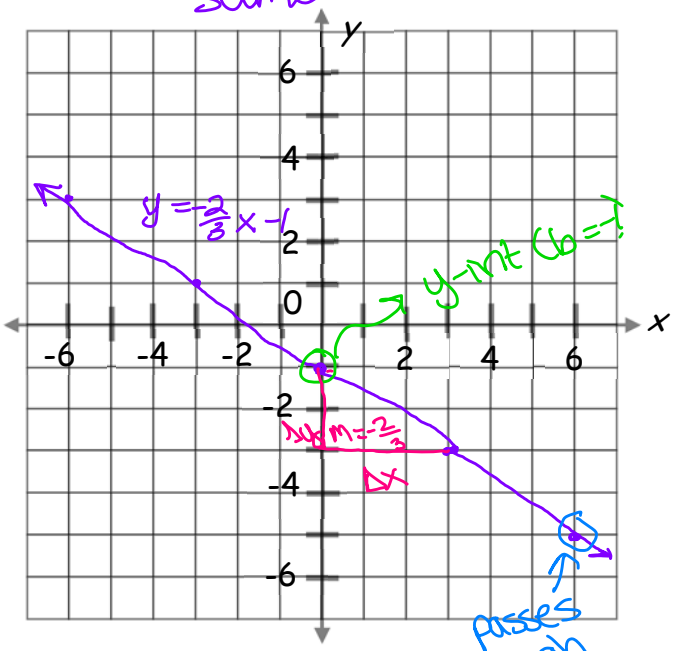
$y - 1 = 2x - 6$

$+1 \quad +1$

$y = 2x - 5$

★ You can solve either way and you'll get the same answer.

6.) Find the equation of a line that has a slope of $-\frac{2}{3}$ and passes through the point (6,-5) using any form you would like.



$y = mx + b$

$y = -\frac{2}{3}x + b$

$-5 = -\frac{2}{3}(6) + b$

$-5 = -4 + b$

$+4 \quad +4$

$-1 = b$

$y = -\frac{2}{3}x - 1$

$y - y_1 = m(x - x_1)$

$y - (-5) = -\frac{2}{3}(x - 6)$

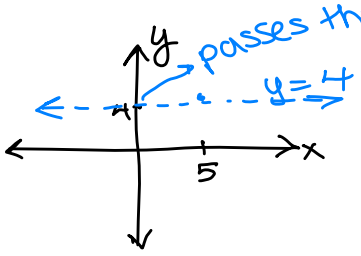
$y + 5 = -\frac{2}{3}x + 4$

$-5 \quad -5$

$y = -\frac{2}{3}x - 1$

7.) Find the equation of a line that has a slope of 0 and passes through the point (5,4).

horizontal line



★ all coordinates have y-values equal to 4. (y=4)

$y = mx + b$

$y = 0x + b$

$4 = 0(5) + b$

$4 = 0 + b$

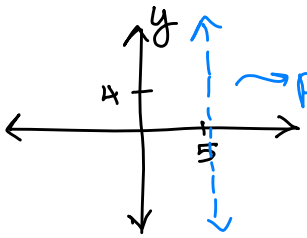
$4 = b$

$y = 0x + 4$

$y = 4$

8.) Find the equation of a line that has an undefined slope and passes through the point (5,4).

vertical line



All coordinates have x-values equal to 5

$x = 5$

Notes 4.6 – Finding Equations of Lines Day 2

1.) Find the equation of a line that passes through (-4,7) and (4,5). Graph the line.

Step 1: Find slope.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-7}{4-4} = \frac{-2}{8} = \frac{-1}{4}$

Step 2: Find y-int. (Use either point)

$y = mx + b$

$y = -\frac{1}{4}x + b$ (Use 4,5)

$5 = -\frac{1}{4}(4) + b$

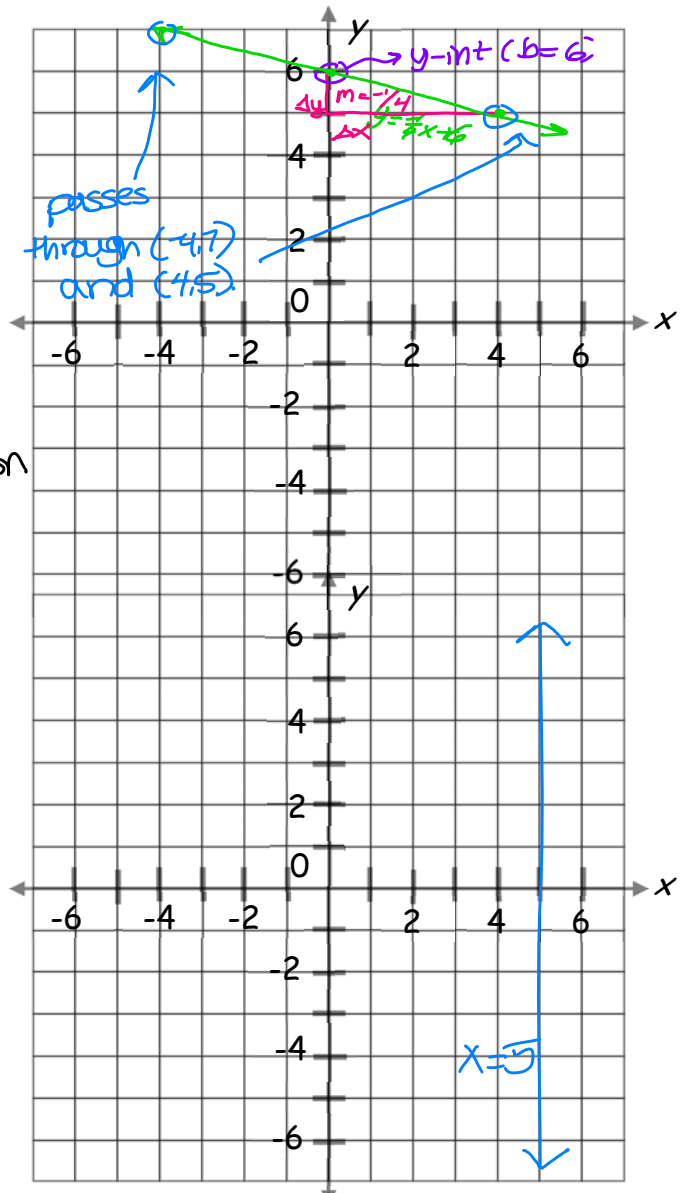
$5 = -1 + b$

$+1 \quad +1$

$6 = b$

Step 3: Write equation

$y = -\frac{1}{4}x + 6$



2.) Find the equation of a line that passes through (5,8) and (5,1). Graph the line.

Step 1: Find slope

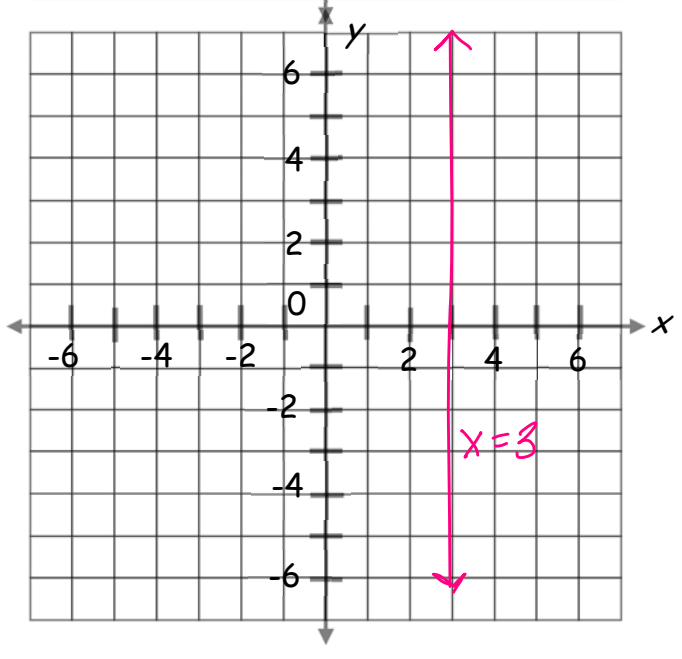
$m = \frac{1-8}{5-5} = \frac{-7}{0}$ = undefined

★ vertical line.

All x-values are equal to 5.

So x will always equal 5.

$x = 5$



3.) Find the equation of a line that passes through (3,1) and (3,8). Graph the line.

$m = \frac{8-1}{3-3} = \frac{7}{0}$ = undefined

★ Vertical line.

All points have x-values of 3.

$x = 3$

Every linear relationship can be represented in four ways: in words, in a table, in a graph, and in an equation. In each of the following examples, you are given one representation and you must find the remaining three representations.

Scenario #1

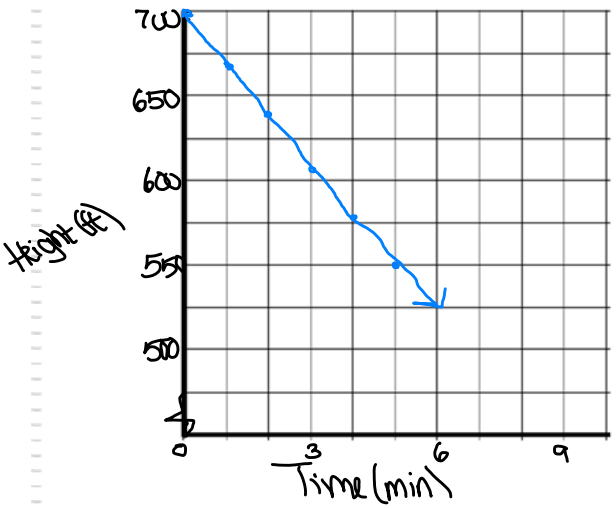
In Words

A cable car starts at a height of 700 feet and is descending at a rate of 30 feet per minute.

In a Table

Time (min)	Height (ft)
0	700
1	670
2	640
3	610

In a Graph



In an Equation

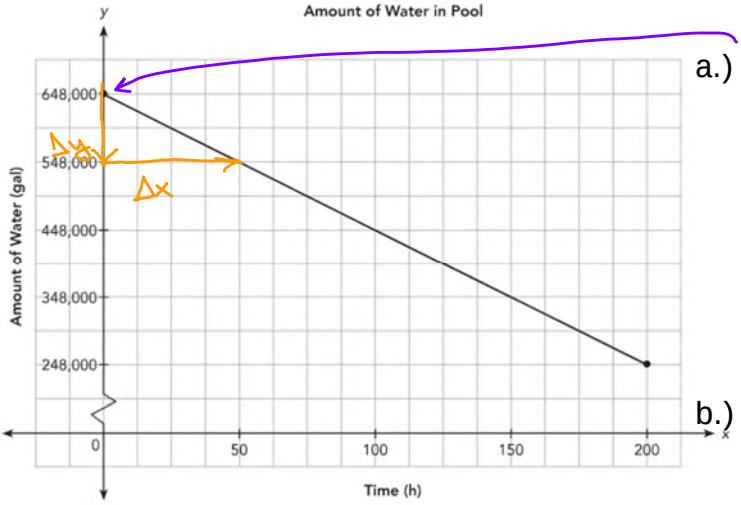
$m = -30$ (decreases by 30 ft/min)

$b = 700$ (initial height)

$y = -30x + 700$

Notes 4.7 – Real-World Applications: Linear Equations

1.) A swimming pool when full holds a certain amount of water. When the drain is opened, the amount of water in the pool drains out at a constant rate. The graph shows the amount of water, W gallons, in the pool h hours after the drain is opened.



a.) Find the vertical intercept of the graph and explain what information it gives about the situation.

(y-int)

$b = 648000$ g

The tank initially had 648,000 gallons of gas.

b.) Find the slope of the graph and explain what information it gives about the situation.

$m = \frac{\Delta y}{\Delta x} = \frac{-100000}{50} = -2000$

The slope is −2000. It means the pool is draining at a rate of 2000 gallons per hour.

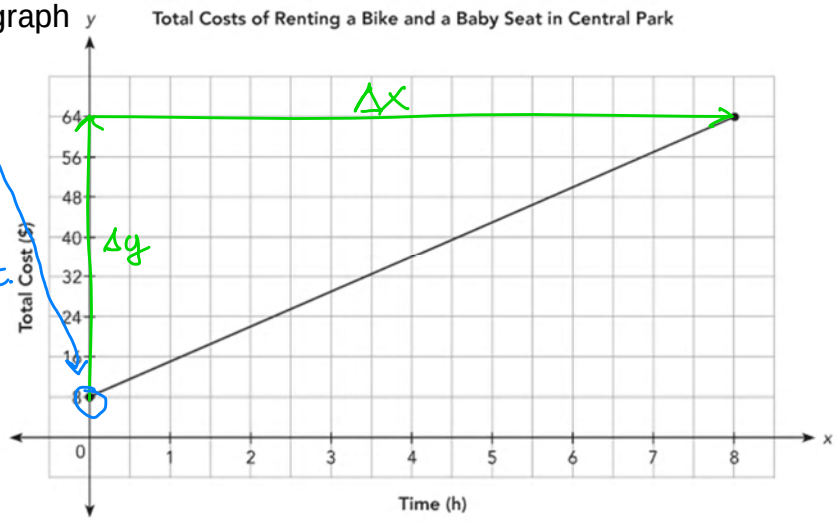
2.) Jeanette rents a bike while visiting a city. She pays $7 per hour to rent the bike. She also pays $8 to rent a baby seat for the bike. She pays this amount for the baby seat no matter how many hours she rents the bike. The graph shows her total cost, C dollars, after h hours.

(m) (b)

a.) Find the vertical intercept of the graph and explain what information it gives about the situation.

$b = 8$

There is an initial cost of $8 to rent a bike w/a baby seat.



b.) Find the slope of the graph and explain what information it gives about the situation.

$m = \frac{\Delta y}{\Delta x} = \frac{56}{8} = \$7/\text{hr.}$

The rate charged to rent a bike is $7/hr.

3.) Anne and Kayla want to join the YMCA. There is a joining fee, plus they need to make monthly payments. After four months, Anne pays a total of $228. After six months, Kayla pays a total of $298.

a.) Write an equation for the cost of joining the YMCA, using *N* for the number of months and *C* for the final cost.

x: independent = time (months) } (4, 228) and (6, 298)
y: dependent = cost (dollars)

$m = \frac{298-228}{6-4} = \frac{70}{2} = \$35/\text{mr}$

$y = mx + b$
$y = 35x + b$

$228 = 35(4) + b$
$228 = 140 + b$
$88 = b$

$y = 35x + 88$
(initial fee is $88 and pay $35/mo)

$C = 35N + 88$

b.) Then, find the total cost of joining the *Y* for 12 months.

$N = 12$
$C = 35N + 88$
$C = 35(12) + 88$
$C = 420 + 88$
$C = \$508$

4.) All the employees of a garden center are given a $0.40 per hour raise each year. You make $7.15 per hour after three years as an employee. Write a linear equation that models your salary per hour, S, in terms of the number of years, N, you have worked at the garden center. Then find your hourly salary after 6 years.

$m = \$0.40/\text{year}$

x: independent: time (yrs)
y: dependent: salary per hr ($) } (3, 7.15)

$y = 0.4x + b$
$7.15 = 0.4(3) + b$
$7.15 = 1.2 + b$
$5.95 = b$

$y = 0.4x + 5.95$
$S = 0.4N + 5.95$

after 6 yrs: $N = 6$
$S = 0.4N + 5.95$
$S = 0.4(6) + 5.95$
$S = 2.4 + 5.95$
$S = \$8.35$