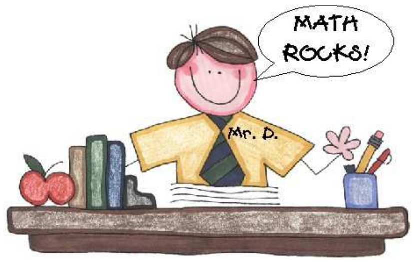
Unit 4 Notes

Systems of Equations



Tentative Schedule

Day	Date	Classwork	Assignment
	Thurs. 10/16 (S) Fri. 10/17 (R)	Test #3	Video #4.1 with Notes – Solving Systems of Equations by Graphing
1	Mon. 10/20 (all)	Problem Set: 1 – 9	Video #4.2/4.3 with Notes – Solving Systems of by Substitution and Elimination
2	Tues. 10/21 (S) Wed. 10/22 (R)	Problem Set: 10 – 29	Video #4.3 with Notes – Applications of Systems of Equations
3	Thurs. 10/23 (all)	**Quiz #4** Problem Set: 30 – 49	Finish Problem Set 30 – 49
4	Tues. 10/28 (all)	Special Cases of Systems of Equations	Complete Unit 4 Problem Sets
5	Wed. 10/29 (S) Thurs. 10/30 (R)	Review for Test #4	Review for Test #4
6	Fri. 10/31 (all)	**Test #4**	

Name: ______________________________________

Notes 4.1 - Solving Systems Graphically

1.) Circle all ordered pairs (x,y) that are solutions to the equation $4x - y = 10$.

x y
(3,2) (2,3) (-1,-14) (0,-10) (3,4)

$4(3)-2=10$
$12-2=10$
$10=10$ ✓

$4(2)-3=10$
$8-3=10$
$5=10$ X

$4(-1)-(-14)=10$
$-4+14=10$
$10=10$ ✓

$4(0)-(-10)=10$
$0+10=10$
$10=10$ ✓

$4(3)-4=10$
$12-4=10$
$8=10$ X

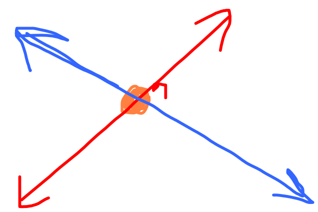
2.) Find another solution to $4x - y = 10$.

$4(5)-10=10$
$20-10=10$

$(5,10)$

3.) How many solutions are there to $4x - y = 10$?

∞

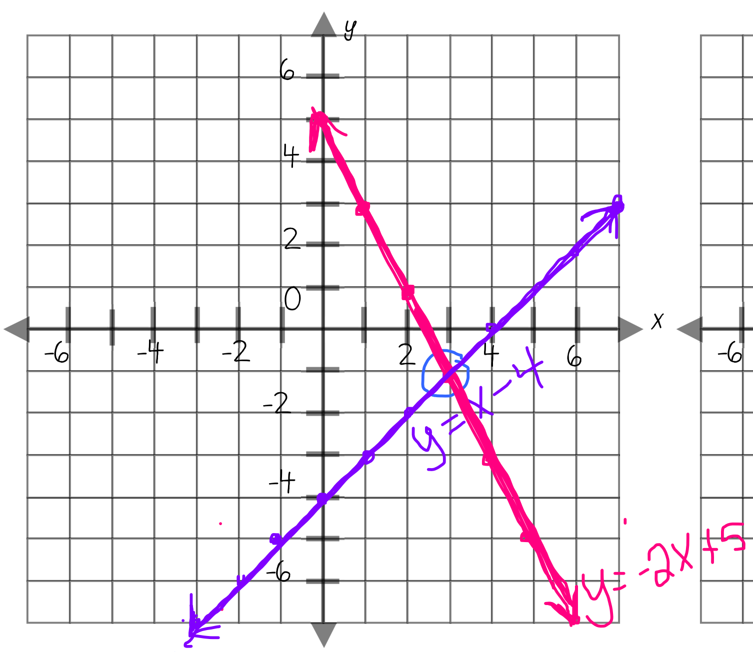


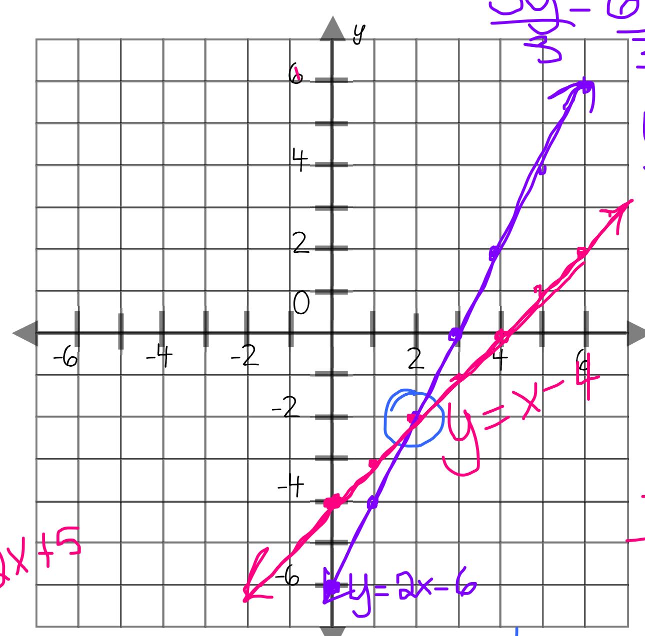
System of Equations: a set of equations that has one simultaneous solution

4.) $y = x - 4$
$y = -2x + 5$

5.) $3y + 18 = 6x$
$x - y = 4$

$3y+18=6x$
$-18 \quad -18$
$\frac{3y}{3}=\frac{6x}{3}-\frac{18}{3}$
$y=2x-6$





$x-y=4$
$+y \quad +y$
$x=4+y$
$-4 \quad -4$
$x-4=y$

$\boxed{(3,-1)}$

Check:
$y=x-4$
$-1=3-4$
$-1=-1$ ✓

$y=-2x+5$
$-1=-2(3)+5$
$-1=-6+5$
$-1=-1$ ✓

$\boxed{(2,-2)}$

6.) Two cyclists are traveling along a track in the same direction. Their motions are described by the linear equations $d = 10t$ and $d - 8t = 2$, where t hours is the time and d miles is the distance from point A on the track.

a.) Solve the system of linear equations using the graphing calculator.

$d = 10t$
$y = 10x (+0)$

$d - 8t = 2$
$d = 8t + 2$
$y = 8x + 2$

$(1, 10)$
After 1 hr, they have the same distance of 10 mi.

b.) When will the cyclists meet?

At the 1 hr point.

7.) All the employees of a garden center are given a \$0.40 per hour raise each year. You make \$7.15 per hour after three years as an employee. Write a linear equation that models your salary per hour, S, in terms of the number of years, N, you have worked at the garden center. Then find your hourly salary after 6 years.

m = \$0.40/yr.
b = ? 5.95

$(3, 7.15)$
$y = .4x + b$
$7.15 = 0.4(3) + b$
$7.15 = 1.2 + b$
$5.95 = b$

$\boxed{y = 0.4x + 5.95}$

$y = 0.4(6) + 5.95$
$y = 2.4 + 5.95$
$\boxed{y = \$8.35/hr}$

Notes 4.2 - Solving Systems Using Substitution

Given the following system of equations, solve for x and solve for y.

$3x - 2y = 4$

$x = 2$

$3x - 2y = 4$

$3(2) - 2y = 4$

$6 - 2y = 4$

$-2y = -2$

$y = 1$

$\boxed{(2,1)}$

Solve the following system of equations.

$y = 3x$

$2x + 5y = 34$

$2x + 5y = 34$

$2x + 5(3x) = 34$

$2x + 15x = 34$

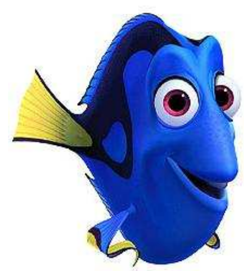
$17x = 34$

$x = 2$

$y = 3(2)$

$y = 6$

$\boxed{(2,6)}$



Solve the following system of equations.

$x = 2y + 2$

$4x + 3y = 41$

$4x + 3y = 41$

$4(2y + 2) + 3y = 41$

$8y + 8 + 3y = 41$

$11y + 8 = 41$

$11y = 33$

$y = 3$

$x = 2(3) + 2$

$x = 6 + 2$

$x = 8$

$\boxed{(8,3)}$

List of Steps to Solve a System of Equations:

$$3x - 2y = 11$$
$$x + 2y = 9$$

Steps	*Solution*
1.) Isolate a variable in one equation. Look for the easiest variable to isolate!	$x + 2y = 9$ $x = 9 - 2y$
2.) ***Substitute*** that into the other equation.	$3x - 2y = 11$ $3(9 - 2y) - 2y = 11$
3.) Now that you only have one variable in the equation, solve it.	$27 - 6y - 2y = 11$ $27 - 8y = 11$ $-8y = -16$ $y = 2$
4.) Plug the answer into any equation to find the other variable.	$x + 2(2) = 9$ $x + 4 = 9$ $x = 5$
5.) Write your answer as a coordinate.	$\boxed{(5, 2)}$
6.) Check the solution in *both* equations.	$3x - 2y = 11$ $3(5) - 2(2) = 11$ $15 - 4 = 11$ $11 = 11$ ✓ $x + 2y = 9$ $5 + 2(2) = 9$ $5 + 4 = 9$ $9 = 9$ ✓

Notes 4.3 - Solving Systems Using Elimination

- Just like substitution, we want to end up with an equation with only __one__ variable. Using this method, we __eliminate__ a variable by __adding__ the equations.
- Make sure the signs are __opposites__.
- Make sure your variables __line__ __up__ before you add!

$x + 2y = 8$
$x - 2y = 4$

You are going to work with your partners to determine a possible solution to solving the system of equations above. It does not matter if your answer is right or wrong. What matters it that you **persevere and you take risks**.

$$\begin{array}{r} x+2y=8 \\ x-2y=4 \\ \hline 2x=12 \end{array}$$

$\frac{2x}{2} = \frac{12}{2}$

$x=6$

$6+2y=8$
$2y=2$
$y=1$

$\boxed{(6,1)}$

$4x + 3y = -1$
$5x + 4y = 1$

You are going to work with your partners to determine a possible solution to solving the system of equations above. It does not matter if your answer is right or wrong. What matters it that you **persevere and you take risks**.

$4(4x+3y=-1) \rightarrow$
$-3(5x+4y=1) \rightarrow$

$$\begin{array}{r} 16x+12y=-4 \\ -15x-12y=-3 \\ \hline x=-7 \end{array}$$

$4(-7)+3y=-1$
$-28+3y=-1$
$3y=27$
$y=9$

$\boxed{(-7,9)}$

1.) $-1(x + y = 18)$ $\quad -\cancel{x} - y = -18$

$x + 2y = 25$ $\quad + \;\; \cancel{x} + 2y = 25$

$y = 7$

$X + y = 18$

$X + 7 = 18$

$X = 11$

$\boxed{(11,7)}$

2.) $3x - 5y = 3$

$+ \; 4x + 5y = 4$

$7x = 7$

$\frac{7x}{7} = \frac{7}{7}$

$X = 1$

$4x + 5y = 4$

$4(1) + 5y = 4$

$4 + 5y = 4$

$5y = 0$

$y = 0$

$\boxed{(1,0)}$

3.) $9(x + 1y = 14)$ $\quad 9x + 9y = 126$

$9x - 9y = 36$ $\quad 9x - 9y = 36$

$18y = 162$

$\frac{18y}{18} = \frac{162}{18}$

$y = 9$

$X + y = 14$

$X + 9 = 14$

$X = 5$

$\boxed{(5,9)}$

4.) $+2x \;\; 3y = -2x + 5$ $\quad 5(2x + 3y = 5)$

$+2x$

$5x + 4y = 16$ $\quad -2(5x + 4y = 16)$

$10x + 15y = 25$

$+ \; -10x - 8y = -32$

$7y = -7$

$y = -1$

$5x + 4y = 16$

$5x + 4(-1) = 16$

$5x - 4 = 16$

$5x = 20 \rightarrow x = 4$

$\boxed{(4,-1)}$

5.) At a baseball game, the players consume 193 gallons of water when the temperature is 50°. When the temperature is 60°, they consume 233 gallons of water.

a. Write a linear function to model the relationship between gallons of water consumed and the temperature.

$(50, 193)$ $(60, 233)$

$m = \frac{233 - 193}{60 - 50} = \frac{40}{10} = 4$ g/degree

$y = 4x + b$

$233 = 4(60) + b$

$233 = 240 + b \rightarrow b = -7$

$\boxed{y = 4x - 7}$

b. Explain the meaning of the slope in the context of the problem

Every time the temp. increases by 1°, the team needs 4 g. of water.

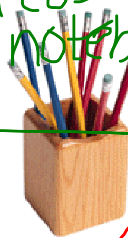
Notes 4.4 - Systems of Equations with Word Problems

Do not forget to write let statements!

1.) Alexa purchased 12 pens and 14 notebooks for \$20. Hannah bought 7 pens and 4 notebooks for \$7.50. Find the price of one pen and the price of one notebook, algebraically.

Let the price of pen = p = \$0.50
Let the price of notebook = n = \$1

One pen costs 50¢ and a notebook is \$1



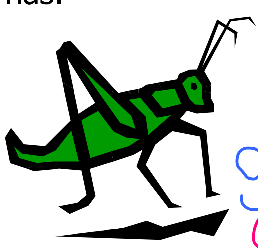
$2(12p + 14n = 20)$ → $24p + 28n = 40$

$-7(7p + 4n = 7.50)$ → $-49p - 28n = -52.50$

$\frac{-25p}{-25} = \frac{-12.50}{-25}$ → $p = \$0.50$

$12(.5) + 14n = 20$
$6 + 14n = 20$
$14n = 14$
$n = 1$

2.) Tyler has a collection of grasshoppers and crickets. He has 561 insects in all. The number of grasshoppers is twice the number of crickets. Find the number of *each* type of insect that he has.



Let the # of grasshoppers = g = 374
Let the # of crickets = c = 187

$g + c = 561$ → $2c + c = 561$
$g = 2c$

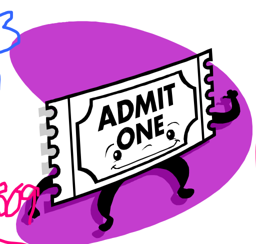
$3c = 561$
$c = 187$

$g = 2(187)$
$g = 374$

There were 374 grasshoppers and 187 crickets

3.) A total of 600 tickets were sold for a concert. If the tickets sold in advance cost \$25 each and the tickets sold at the door cost \$32 each, and \$16,309 worth of tickets was sold, how many of each type of ticket was sold?

Let the # of advance tix = a = 413
Let the # of door tix = d = 187



$a + d = 600$
$d = 600 - a$

$25a + 32d = 16309$
$25a + 32(600 - a) = 16309$
$25a + 19200 - 32a = 16309$
$-7a + 19200 = 16309$
$-7a = -2891$
$a = 413$

$a + d = 600$
$413 + d = 600$
$d = 187$

There were 413 advance tickets and 187 door ticket sold.

Notes 4.5 - Special Cases of Systems of Equations

Warm-up: Please solve the following equations.

You have learned to find the unique solution to a system of linear equations, when it exists. However, not every system of linear equations has a unique solution.

1.) $3(x+4)=2x+17+x-5$

$3x+12=3x+12$

∞ solutions

2.) $2(x+5)+3x=5x+14$

$2x+10+3x=5x+14$

$5x+10=5x+14$

$10\neq 14$

no solution

3.) With your partners, please solve the following system of equations using *substitution*.

$2x+y=1$

$4x+2y=4$

$y=1-2x$

$4x+2(1-2x)=4$

$4x+2-4x=4$

$2\neq 4$

No solution

4.) With your partners, please solve the following system of equations using *elimination.*

$-2(2x+y=1) \rightarrow -4x-2y=-2$

$4x+2y=4 \rightarrow 4x+2y=4$

$0=2$

No solution

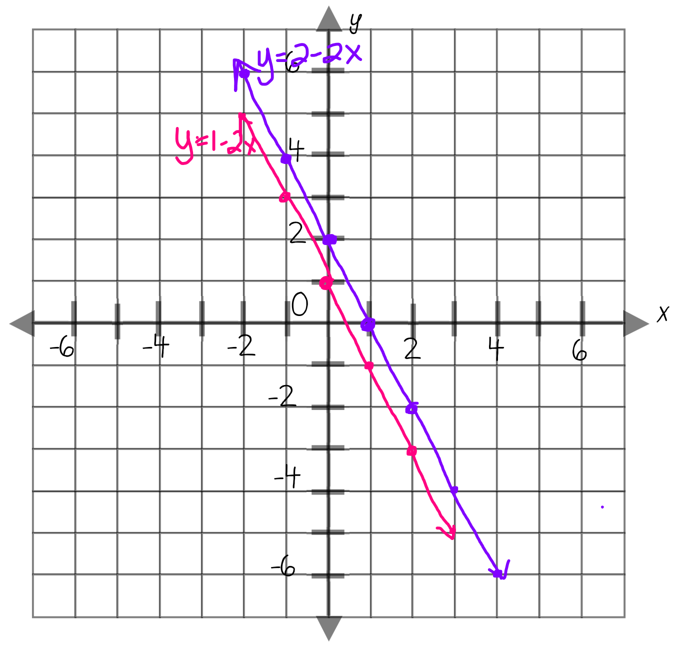
5.) With your partners, please solve the following system of equations by *graphing*.

$2x + y = 1 \rightarrow y = 1 - 2x$

$4x + 2y = 4$

$2y = 4 - 4x$

$y = 2 - 2x$



No solution

6.) What happened when you tried to solve the equation with all three methods?!

They all had no solution.

7.) With your partners write a **thorough explanation** why this happened algebraically.

When simplified, each equation has the same coefficients for each variable and different constants.

8.) With your partners write a **thorough explanation** why this happened graphically.

The lines have the same slope and have different y-intercepts making the lines parallel. Thus, they never intersect, therefore there is no solution.

9.) With your partners, please solve the following system of equations using ***substitution***.

$x + 2y = 2$
$2x + 4y = 4$

$x = 2 - 2y$
$2x + 4y = 4$
$2(2 - 2y) + 4y = 4$
$4 - 4y + 4y = 4$
$4 = 4$

$\boxed{\infty \text{ solutions}}$

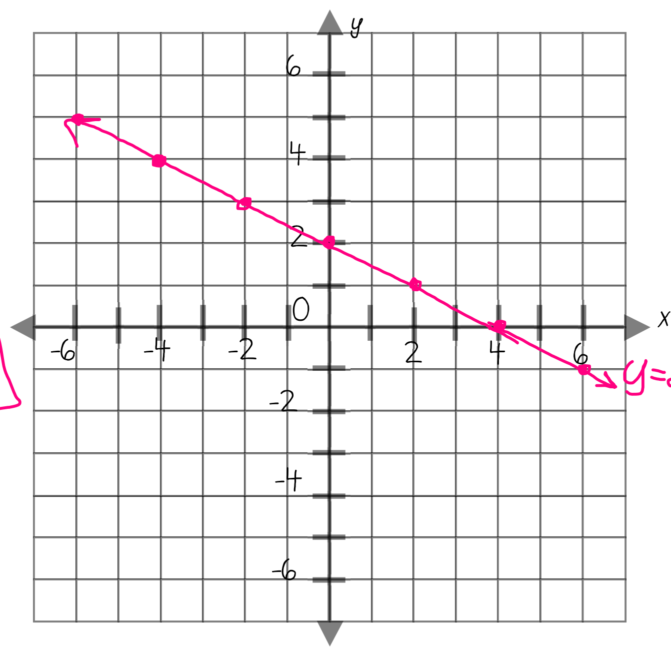
10.) With your partners, please solve the following system of equations by ***graphing***.

$x + 2y = 2$
$2x + 4y = 4$

$x + 2y = 2$
$2y = 2 - x$
$y = 1 - \frac{1}{2}x$

$2x + 4y = 4$
$4y = 4 - 2x$
$y = 1 - \frac{1}{2}x$

$\boxed{\infty \text{ solutions}}$



11.) With your partners, please solve the following system of equations using ***elimination.***

$-2(x + 2y = 2) \rightarrow -2x - 4y = 4$
$2x + 4y = 4 \rightarrow 2x + 4y = 4$

$0 = 0$

$\boxed{\infty \text{ solutions}}$

12.) What happened when you tried to solve the equation with all three methods?!

There were infinite solutions

13.) With your partners write a **thorough explanation** why this happened with algebraically.

When simplified, the equations were identical. Therefore, there were infinite solutions.

14.) With your partners write a **thorough explanation** why this happened graphically.

The lines had the same slope and the same y-intercepts. Thus, they are the same line and will intersect at all points. Therefore, there are infinite solutions.

Summary:

- There is no solution when the coefficients are the same but the constants are different, and when the lines are parallel.
  (when the equations are simplified)

  Example:

  $3x+6y=15$
  $x+2y=4$

- There is one unique solution when the coefficients of the simplified equation are different for at least one variable and when the lines intersect.

  Example:

  $5x+2y=7$
  $8x+2y=4$

- There are infinite solutions when the simplified equations are identical and when they are the same line.

  Example:

  $4x+7y=3$
  $8x+14y=6$