Unit 5 Notes

Inequalities and Absolute Value

$a > b \Rightarrow b < a$

"Reversal" *Property of Inequality*

Tentative Schedule

Day	*Date*	*Classwork*	*Assignment*
	Fri. 10/31 (all)	Test #4	Video #5.1 with Notes: Inequalities in One Variable
1	Mon. 11/3 (S) Tues. 11/4 (R)	1 – 9	Video #5.2 with Notes: Compound Inequalities
2	Mon. 11/10 (all)	10 – 26	Video #5.3 with Notes: Two-Variable Inequalities
3	Wed. 11/12 (S) Thurs. 11/13 (R)	27 – 36	Video #5.4 with Notes: Systems of Inequalities
4	Fri. 11/14 (all)	37 – 45	Video #5.5 with Notes: Absolute Value Equations
5	Mon. 11/17 (S) Tues. 11/18 (R)	46 – 63	Video #5.6 with Notes: Absolute Value Inequalities
6	Wed. 11/19 (all)	64 – 74	Finish Problem Sets
7	Thurs. 11/20 (S) Fri. 11/21 (R)	Review for Test #5	Review for Test #5
8	Mon. 11/24 (all)	**Test #5**	Video #6.1 with Notes

Name: ______________________________________

Notes 5.1 - One-Variable Inequalities

Inequality Symbols

$a < b$: a is less than b

$a \le b$: a is less than or equal to b

$a > b$: a is greater than b

$a \ge b$: a is greater than or equal to b

Graphing Inequalities

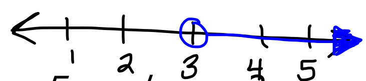
1.) Always draw a number line.

2.) Draw a circle around the number.

a.) Use an open circle for < or > statements.

b.) Use a closed circle for ≤ or ≥ statements.

3.) Shade the number line in the correct place.

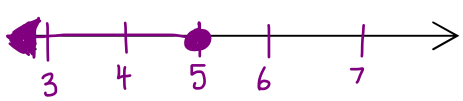


Example: $x > 3$

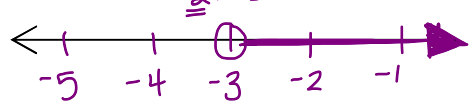
What is the point of drawing a number line?

In an inequality, the solution set has infinite elements. It's impossible to list all elements. Makes more sense to represent graphically

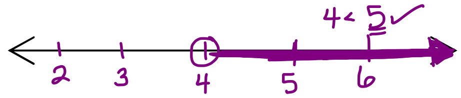
1.) $x \le 5$ Test: $4 \le 5$ ✓



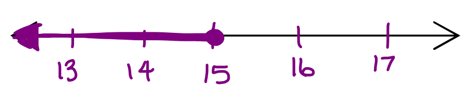
2.) $x > -3$ Test: $-2 > -3$ ✓



3.) $4 < x \leftrightarrow x > 4$ Test: $4 < 5$ ✓



4.) x is **at most** 15. $x \le 15$

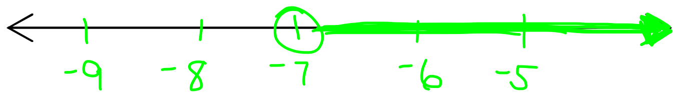


Test:
$-6z<42$
$-6(-5)<42$
$30<42$ ✓

5.) $-6z < 42$

$\frac{-6z}{-6} < \frac{42}{-6}$

$z > -7$



Important!

If you have to multiply or divide by a negative number, you have to ALWAYS change the inequality sign!!!

Why?

The negatives are reversed.

6.) $5x - 7 \le x + 9$

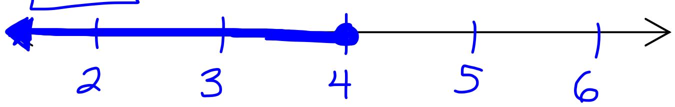
$-x \quad -x$

$4x-7\le 9$

$+7 \quad +7$

$\frac{4x}{4}\le \frac{16}{4}$

$\boxed{x\le 4}$



7.) If three-fourths of a whole number decreased by 8 is at least 3, what is the smallest number that will satisfy the solution?

Let whole # $= x$

$\frac{3}{4}x-8\ge 3$

$+8 \quad +8$

$\cancel{4}\cdot\frac{3}{\cancel{4}}x\ge 11\cdot 4$

$\frac{3x}{3}\ge \frac{44}{3}$

$x\ge 14\frac{2}{3}$

The smallest whole # greater than $14\frac{2}{3}$ is $\boxed{15}$

8.) Stephen decided that he would spend at most \$450 on a snowboard and a helmet with speakers. If the price of the snowboard was \$100 less than four times the price of the helmet, find the highest possible price of the snowboard.

Let price of helmet $= x$

Let price of snowboard $= 4x-100$

$x+4x-100\le 450$

$5x-100\le 450$

$5x\le 550$

$x\le 110$

Highest value x can be is \$110.

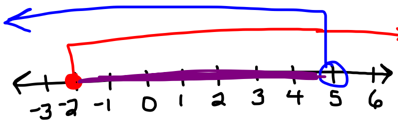
Snowboard:
$4x-100=$
$4(110)-100=440-100=\$340$

max helmet price $=\$110$.
max snowboard price $=\$340$

Notes 5.2 - Double Inequalities

1.) Graph the solution set of $x \geq -2$ and $x < 5$.

must satisfy both (where red + blue overlap)



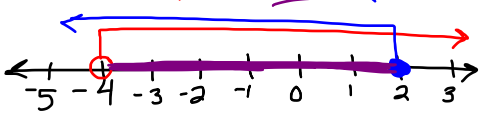
2.) Graph the solution set of $-2 \leq x < 5$.

$-2 \leq x$ and $x \leq 5$

EXACTLY LIKE #1

3.) Graph the solution set of $-4 < x \leq 2$.

$-4 < x$ and $x \leq 2$

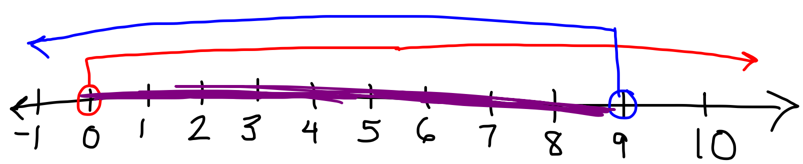


4.) Write the compound inequality without using and. Then graph the solution set.

$x < 9$ and $x > 0$.

$0 < x$ and $x < 9$

$\boxed{0 < x < 9}$

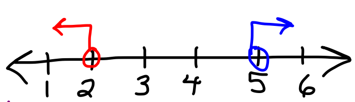


5.) Graph the solution set of $x \geq 1$ or $x < -4$.

can satisfy one or both (doesn't have to overlap)

6.) Graph the solution set of $x > 5$ and $x < 2$.

must satisfy both (must overlap)

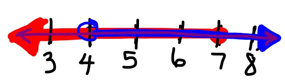


Both conditions aren't satisfied, so $\boxed{\text{NO SOLUTION}}$

(Think about it logically... Can you find a # that is both less than 2 and greater than 5? Impossible!)

7.) Graph the solution set of $x \leq 7$ or $x > 4$.

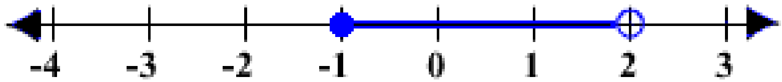
no overlap necessary.



$\boxed{\text{X can be any real \#.}}$

(Any # is either less than 7 or greater than 4)

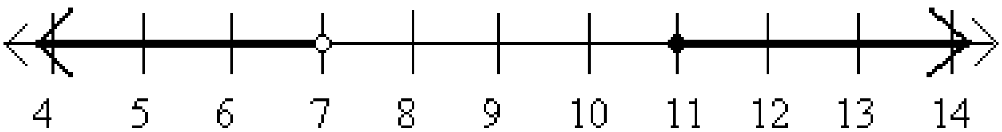
8.) Write a compound inequality for the solution set shown below.



$x \geq -1$ and $x < 2$

$-1 \leq x$ and $x < 2 \rightarrow \boxed{-1 \leq x < 2}$

9.) Write a compound inequality for the solution set shown below.



$\boxed{x < 7 \quad \text{or} \quad x \geq 11}$

Solve the following compound inequalities. Then, graph the solution set.

10.) $20 < -3x + 11 \leq 29$

$20 < -3x + 11$ and $-3x + 11 \leq 29$

$-11 \quad -11 \qquad\qquad -11 \quad -11$

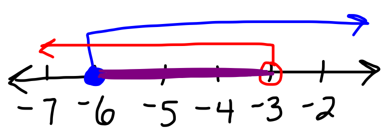
$9 < -3x \qquad\qquad -3x \leq 18$

$\frac{9}{-3} < \frac{-3x}{-3} \qquad\qquad \frac{-3x}{-3} \leq \frac{18}{-3}$

$-3 > x$ and $x \geq -6$

$x < -3$ and $-6 \leq x$

$\boxed{-6 \leq x < -3}$

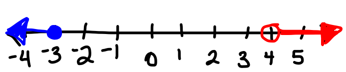


11.) $5m - 7 > 13$ or $5m - 7 \leq -22$

$+7 \quad +7 \qquad\qquad +7 \quad +7$

$\frac{5m}{5} > \frac{20}{5} \qquad\qquad \frac{5m}{5} \leq \frac{-15}{5}$

$\boxed{m > 4 \quad \text{or} \quad m \leq -3}$



Notes 5.3 - Graphing Inequalities in Two-Variables

Step 1: Graph the inequality as if you would graph the line.

Step 2: If the sign is < or >, use a dotted line.

If the sign is ≤ or ≥, use a Solid line.

Step 3: Choose a point on the graph and plug it in to see if you should shade in that region.

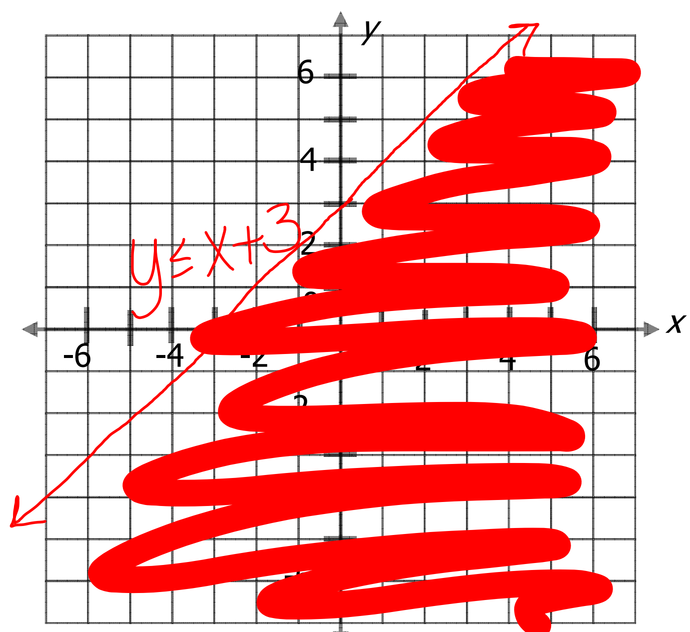
1.) $y \le x + 3$

Test (0,0)

$0 \le 0 + 3$

$0 \le 3$ ✓

True – shade on side that has (0,0).



2.) $2y > 5x - 4$

$\frac{2y}{2} > \frac{5x}{2} - \frac{4}{2}$

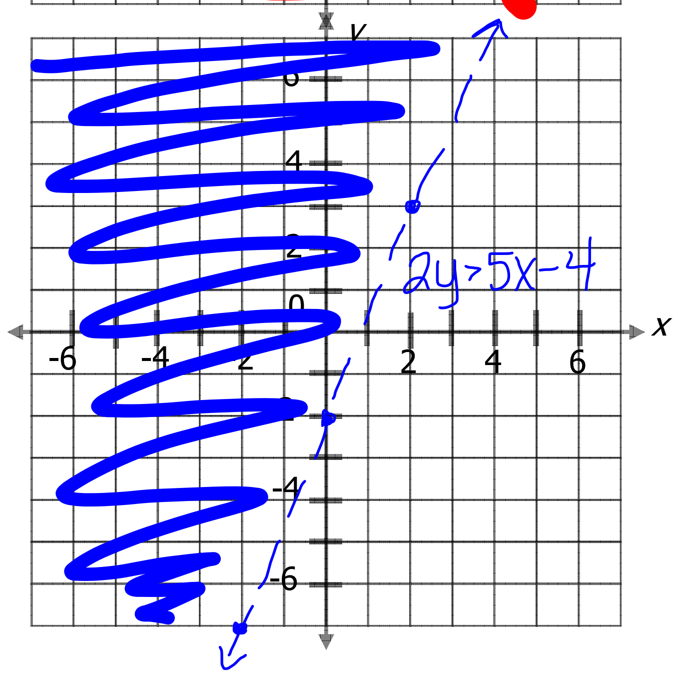
$y > \frac{5}{2}x - 2$

Test (0,0)

$2(0) > 5(0) - 4$

$0 > -4$

True

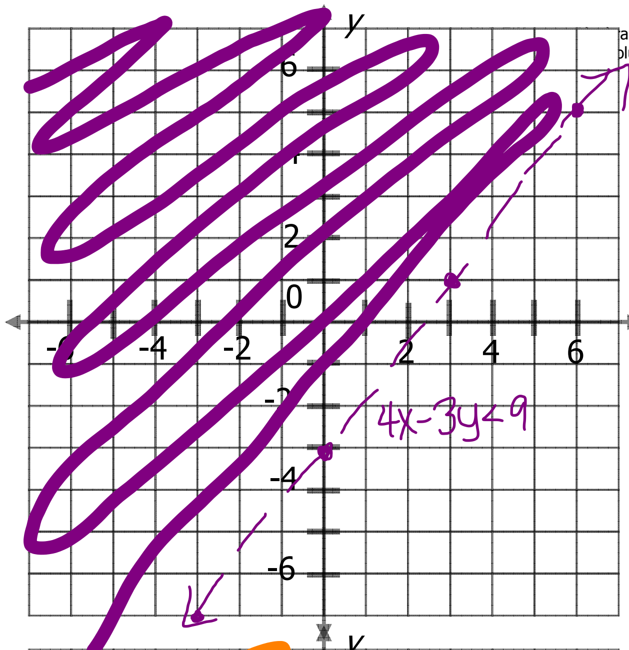


3.) $4x - 3y < 9$

$4x - 3y < 9$
$+3y \quad +3y$
$4x < 3y + 9$
$-9 \quad -9$
$\frac{4x}{3} - \frac{9}{3} < \frac{3y}{3}$
$\frac{4}{3}x - 3 < y$
$y > \frac{4}{3}x - 3$

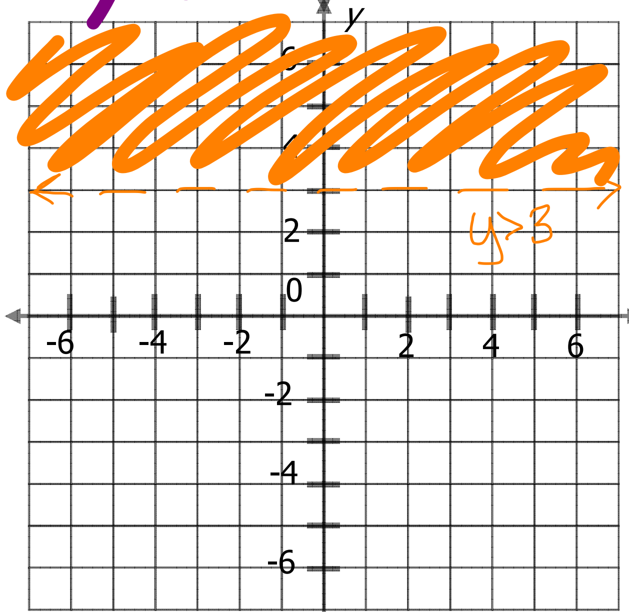
$4x - 3y < 9$
$-4x \quad -4x$
$\frac{-3y}{-3} < \frac{-4x}{-3} + \frac{9}{-3}$
$y \boxed{>} \frac{4}{3}x - 3$

Test: (0,0)
$4(0) - 3(0) < 9$
$0 < 9$ ✓



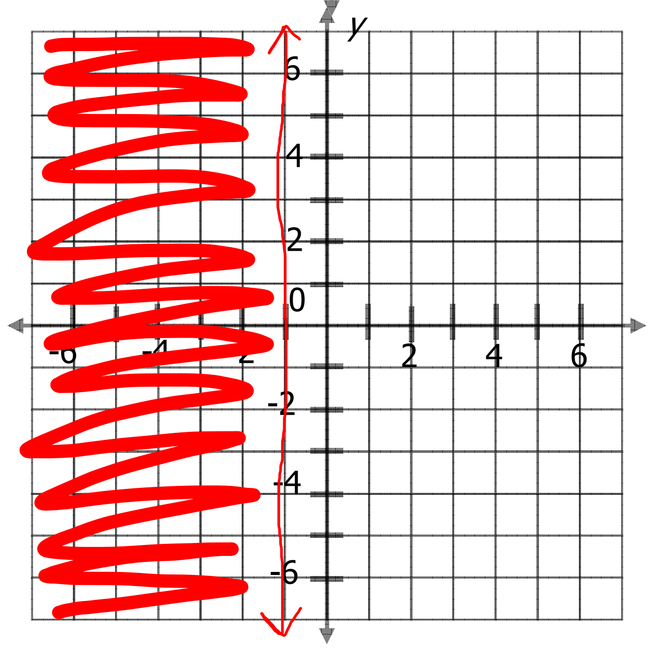
4.) $y > 3$

y=3 is a horizontal line
Test (0,0)
$0 > 3$ X
Not true



5.) $x \le -1$

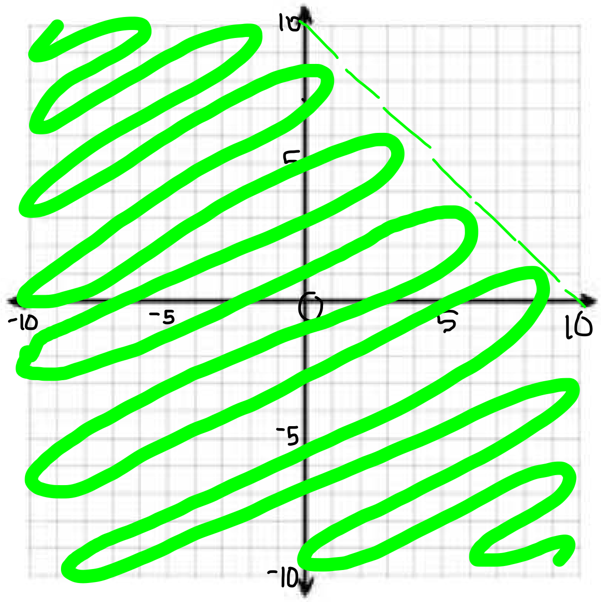
X=-1 is a vertical line
(0,0) Test
$0 \le -1$ X
Not true



6.) What pairs of numbers satisfy the statement: The sum of two numbers is less than 10? Create an inequality with two variables to represent this situation and graph the solution set.

$x + y < 10$

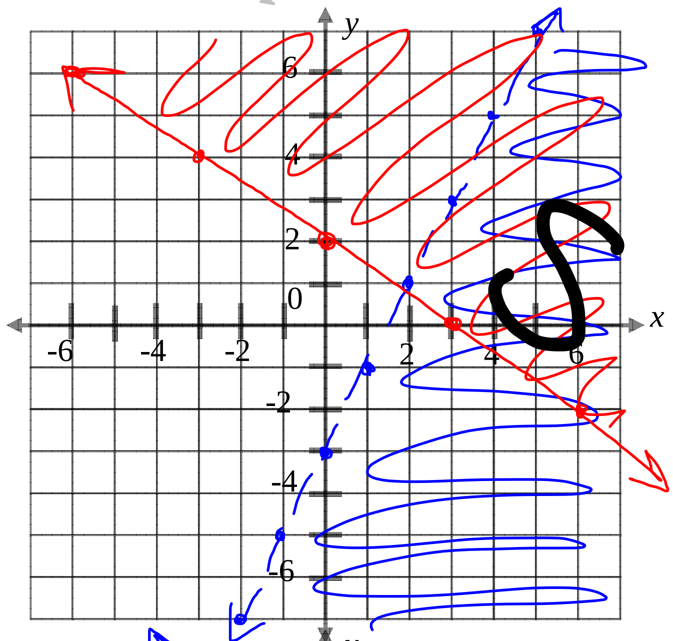
$y < 10 - x$



Notes 5.4 - Systems of Inequalities

1.) $y < 2x - 3$

$y \geq -\frac{2}{3}x + 2$

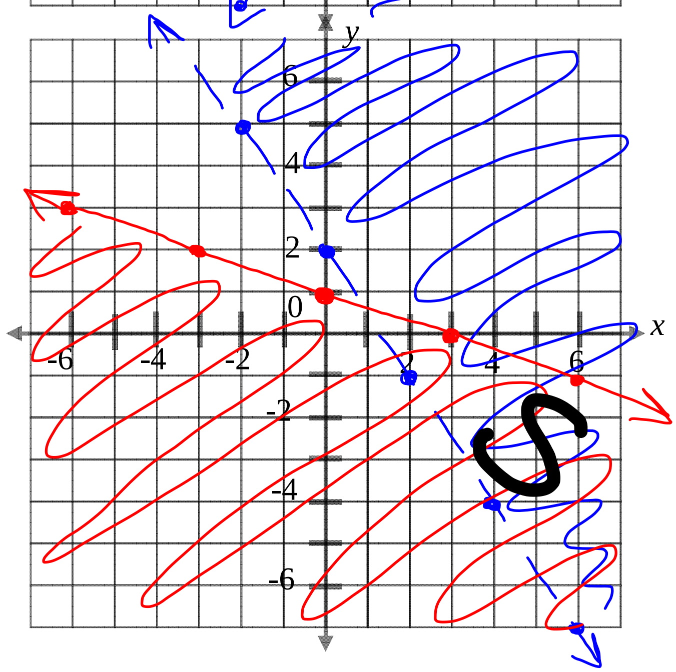


What is one point that will satisfy the solution? (6,1)

(Answers vary)

2.) $-2y < 3x - 4$

$3y + x \leq 3$



$y \leq -\frac{1}{3}x + 1$

What is one point that will satisfy the solution? (6,-3)

$$\frac{-2y}{-2} < \frac{3x}{-2} - \frac{4}{-2}$$

$$y > -\frac{3}{2}x + 2$$

$$3y + x \leq 3$$
$$-x \quad -x$$

$$\frac{3y}{3} \leq \frac{-x}{3} + \frac{3}{3}$$

$$y \leq -\frac{1}{3}x + 1$$

3.) A clothing manufacturer has 1000 yd. of cotton to make shirts and pajamas. A shirt requires 1 yd. of fabric and a pair of pajamas requires 2 yd. of fabric. It takes 2 hr. to make a shirt and 3 hr. to make the pajamas, and there are 1600 hr. available to make the clothing.

a.) What are the variables?

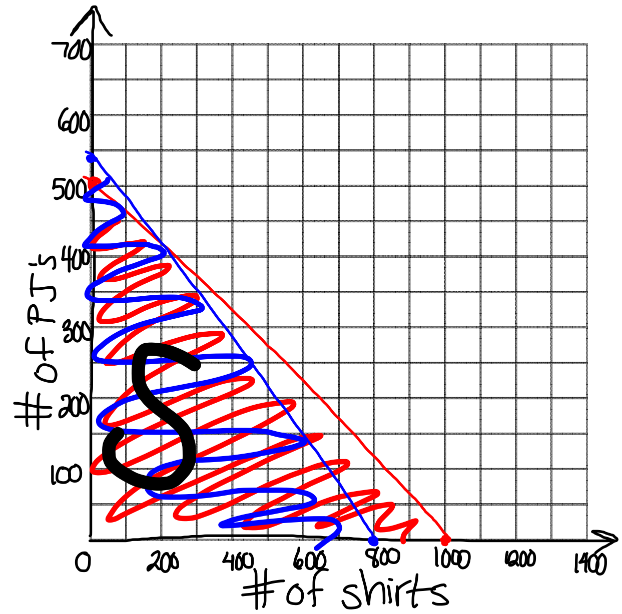
# of shirts = x
# of PJ's = y

b.) What are the constraints?

1000 yd. of fabric
1600 hr. of manufacturing

c.) Write inequalities for the constraints.

$1x + 2y \leq 1000$
$2x + 3y \leq 1600$



d.) Graph the inequalities and shade the solution set.

Find x- and y-intercepts

$1x + 2y \leq 1000$
$1(0) + 2y \leq 1000$
$2y \leq 1000$
$y \leq 500$
(0,500) - y int
$1x + 2(0) \leq 1000$
$1x \leq 1000$
(1000,0) - x int

$2x + 3y \leq 1600$
$2(0) + 3y \leq 1600$
$3y \leq 1600$
$y \leq 533\frac{1}{3}$
$(0, 533\frac{1}{3})$ y-int

$2x + 3(0) \leq 1600$
$2x \leq 1600$
$x \leq 800$
(800,0) - x int

e.) What does the shaded region represent?

All the possible combinations of shirts & PJ's that are work under the given constraints

f.) Suppose he makes a profit of $10 on shirts and $18 on pajamas. How would he decide how many of each to make?

He wants to make as many as possible so the maximum should be at one of the endpoints of the shaded region

g.) How many of each should he make assuming he will sell all the shirts and pajamas he makes? Profit = 10x + 18y

Possible Points
(0,500): P = 10(0) + 18(500) = $9,000
(200,400): P = 10(200) + 18(400) = $9,200
(800,0): P = 10(800) + 18(0) = $8000

$\boxed{\text{200 shirts, 400 PJ's}}$

Notes 5.5 - Solving Equations Involving Absolute Value

Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable. For example, if a survey on the reading habits of people in the US resulted in 46% of people reading popular fiction, with an error of $\pm 3\%$, what percent of people could read popular fiction?

$46-3=43\%$
$46+3=49\%$

anywhere between 43% and 49%

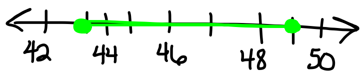
1.) Evaluate $|m+6|-14$ if m = 4.

$|4+6|-14$
$|10|-14$
$10-14=\boxed{-4}$

2.) Evaluate $23-|3-4x|$ if x = 2.

$23-|3-4(2)|$
$23-|3-8|$
$23-|-5|$
$23-5=\boxed{18}$

3.) The margin of error in the example at the top of the page is an example of absolute value. Graphically represent the percentage of people that read popular fiction.



4.) Solve for x: $|x|=4$

$x=4$
or
$x=-4$

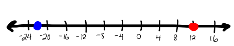
$\boxed{\{-4,4\}}$

Solve each equation. Then, graph the solution set.

5.) $|f+5|=17$

$f+5=17$ OR $f+5=-17$
$-5\ \ -5$ $\quad -5\ \ -5$
$f=12$ $\quad f=-22$

$\boxed{\{12,-22\}}$



6.) $|b+1|=-3$

No solution!
(No absolute value can equal a negative!)

7.) Write an equation involving absolute value for a solution set of $\{11,19\}$.

$\frac{11+19}{2} = \frac{30}{2} = 15$

Each value is 4 away from 15 (difference, whether positive or negative is 4)

$\boxed{|x-15| = 4}$

8.) Solve: $|2x-3|-4=3$.

$+4 \quad +4$

Isolate absolute value first!

$|2x-3| = 7$

$2x-3=7$ OR $2x-3=-7$

$2x=10$ $\quad$ $2x=-4$

$x=5$ $\quad$ $x=-2$

$\boxed{\{-2, 5\}}$

9.) Solve $|3x+2| = 4x+5$.

$3x+2=4x+5$ OR $3x+2=-(4x+5)$

$-x+2=5$

$-x=3$

$x=-3$

Check:

$|3(-3)+2| = 4(-3)+5$

$|-9+2| = -12+5$

$|-7| = -7$

$7 \neq -7$

$x=-3$ DOESN'T WORK!

$3x+2=-4x-5$

$7x+2=-5$

$7x=-7$

$x=-1$

Check:

$|3(-1)+2| = 4(-1)+5$

$|-3+2| = -4+5$

$|-1| = 1$

$1=1$ ✓

The only solution is $x=1$

Notes 5.6 - Solving Inequalities Involving Absolute Value

1.) Consider the inequality $|x| \le 4$. What values of x satisfy the inequality?

Integer values that work:
$-4, -3, -2, -1, 0, 1, 2, 3, 4 \longrightarrow$ $x \le 4$ and $x \ge -4$

or

$\boxed{-4 \le x \le 4}$

2.) Consider the inequality $|x| \ge 4$. What values of x satisfy the inequality?

Integer values that work:
$4, 5, 6, 7, \ldots$
OR
$-4, -5, -6, -7, \ldots$

$\boxed{x \ge 4 \text{ or } x \le -4}$

$<$ or $\le$: and
$>$ or $\ge$: or

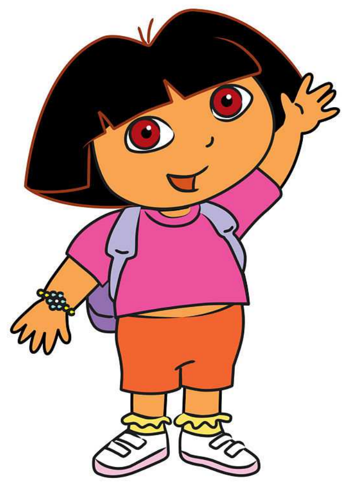
Solve the following inequalities:

3.) $|x-3| \le 5$

$x - 3 \le 5$ AND $x - 3 \ge -5$ must be flipped!
$+3 \quad +3$ $\qquad$ $+3 \quad +3$
$x \le 8$ AND $x \ge -2$

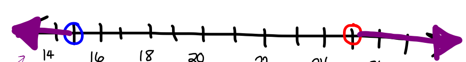
$\boxed{-2 \le x \le 8}$

Test $x=-3$: $|-3-3| < 5$, $|-6| < 5$, $6 < 5$ (No)
Test $x=4$: $|4-3| < 5$, $|1| < 5$, $1 < 5$ (yes)
Test $x=9$: $|9-3| < 5$, $|6| < 5$, $6 < 5$ (No)



4.) $|x-20| > 5$

$x - 20 > 5$ OR $x - 20 < -5$
$+20 \quad +20$ $\qquad$ $+20 \quad +20$
$x > 25$ OR $x < 15$



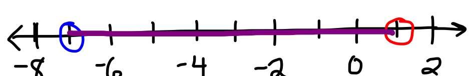
Test $x=13$: $|13-20| > 5$, $|-7| > 5$, $7 > 5$ yes!
Test $x=20$: $|20-20| > 5$, $|0| > 5$, $0 > 5$ No
Test: $x=26$: $|26-20| > 5$, $|6| > 5$, $6 > 5$ ✓ yes!

5.) $|3+x| - 4 < 0$

$|3+x| < 4$

$3 + x < 4$ AND $3 + x > -4$
$-3 \quad -3$ $\qquad$ $-3 \quad -3$
$x < 1$ AND $x > -7$

$\boxed{-7 < x < 1}$



Test: $|3+(-8)| - 4 < 0$, $|-5| - 4 < 0$, $5 - 4 < 0$, $1 < 0$ No
Test $x=-5$: $|3+(-5)| - 4 < 0$, $|-2| - 4 < 0$, $2 - 4 < 0$, $-2 < 0$ yes!
Test $x=3$: $|3+3| - 4 < 0$, $|6| - 4 < 0$, $6 - 4 < 0$, $2 < 0$ No

6.) $5 < |x+1| < 7$

$5 < |x+1|$

$|x+1| > 5$

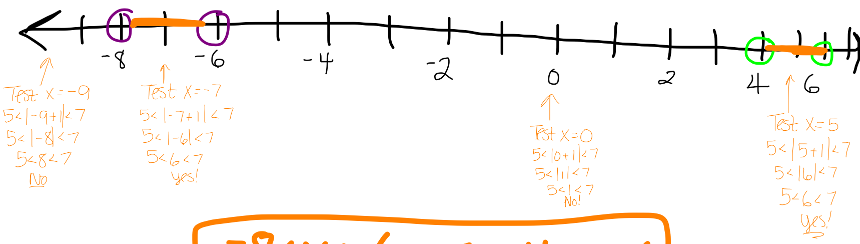
$x+1 > 5$ $\quad$ $x > 4$

$x+1 < -5$ $\quad$ $x < -6$

$|x+1| < 7$

$x+1 < 7$ $\quad$ $x < 6$

$x+1 > -7$ $\quad$ $x > -8$



$-8 < x < -6$ or $4 < x < 6$

$(-8, -6) \cup (4, 6)$

7.) $|x+4| > -3$

All values of x satisfy this inequality because the absolute value of any number will always be greater than -6

X is any real #

8.) $|x+1| < -6$

No value of x can satisfy this inequality because the absolute value is always non-negative and thus never less than -6.

no solution

9.) At the Brooks Graphic Company, the average starting salary for a new graphic designer is \$37,600, but the actual salary could differ from the average by as much \$2590.

a.) Write an absolute value inequality to describe this situation.

$|x - 37{,}600| < 2590$

b.) Solve the inequality to find the range of the starting salaries.

$x - 37600 < 2590$ AND $x - 37600 > -2590$

$+37600 \quad +37600$ $\qquad$ $+37600 \quad +37600$

$x < 40190$ AND $x > 35010$

$35010 < x < 40190$