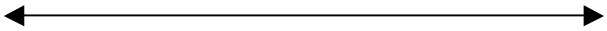
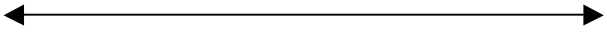
Unit 5 Problem Sets - Inequalities and Absolute Value

Name: _____ Class: _____

PS #5.1 – One-Variable Single Inequalities

For 1 – 7, find and graph the solution set of the inequality. Write the answer in interval notation.

Inequality	*Graph*
1.) $7y - 4 < 6 + 2y$	
2.) $8n \geq 5(2n + 4)$	
3.) $-3(4x - 8) \geq 2(3 + 2x)$	

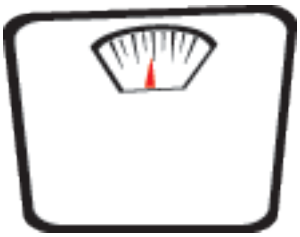
4.) $8m - 2(2m + 3) \geq 0$

For 5 – 8, define a variable, write an inequality, and solve it.

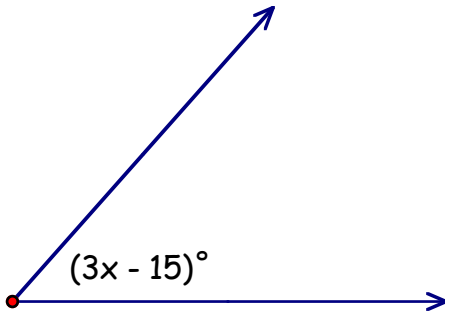
5.) Erica decided that she would spend no more than $120 to buy a jacket and a skirt. If the price of the jacket was $20 more than 3 times the price of the skirt, find the highest possible price of the skirt.

6.) The length of a rectangle is 10 cm less than 3 times the width. If the perimeter of the rectangle is at most 180 cm, find the greatest possible length of the rectangle. (Hint: draw a picture).

7.) Karen weighs 3 times as much as Sue. Both weights are whole numbers. The sum of their weights is less than 160 pounds. Find the greatest possible weight in pounds for each girl.



8.) Determine the value of x so that angle A is acute.



9.) Six more than 4 times a whole number is less than 60. Find the maximum value of the number.

PS #5.2 – One-Variable Compound Inequalities

For 10 – 17, graph the following compound inequalities. Express each answer in interval notation if possible.

10.) $m \geq -5$ and $m < -3$

11.) $p < -2$ and $p > 4$

12.) $s < 3$ or $s \geq 1$

13.) $n \leq -5$ or $n \geq -1$

14.) $w > -3$ and $w < 1$

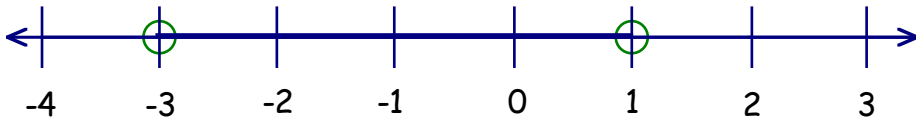
15.) $x < -4$ or $x \geq 0$

16.) $4 < x < 7$

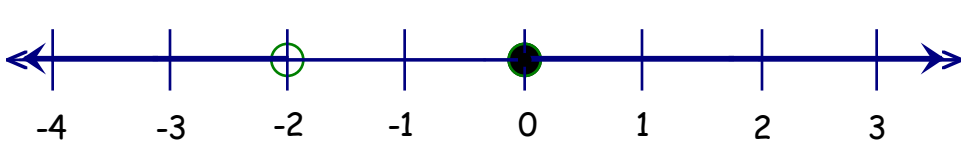
17.) $-3 \leq x < 2$

For 18 – 19, write a compound inequality for each solution set shown below.

18.)



19.)

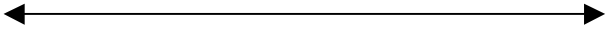


Solve each compound inequality. Then graph the solution set. Write the solution interval notation, if possible.

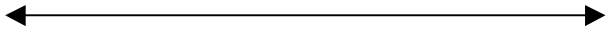
20.) $3n + 2 < -2 + 7n$ or $8n - 4 \leq 3n - 4$

21.) $5w > 4(2w - 3)$ and $5(w - 3) + 2 < 7$

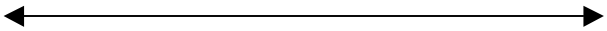
22.) $4 < 2x - 2 \leq 10$



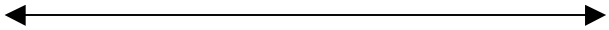
23.) $5x + 7 \geq 2x + 4$ or $3x + 3 < 24 - 4x$



24.) $4z + 9 \geq z + 3$ or $7z - 14 \geq 2z - 4$



25.) $3 + y > 2y > -3 - y$



26.) The height of a horse is measured in a vertical line from the ground to the withers (at the base of the neck). Horses are measured in "hands" where one hand is 4 inches. If a horse is more than an exact number of hands high (hh), the extra inches are given after a decimal point, e.g. 14 hands 2 inches is written as 14.2 hh. Normal riding horses are between 14.3 hh and 17 hh, inclusive. Horses shorter than 14.3 hands are called ponies and horses over 17 hh are often called draft (or work) horses.

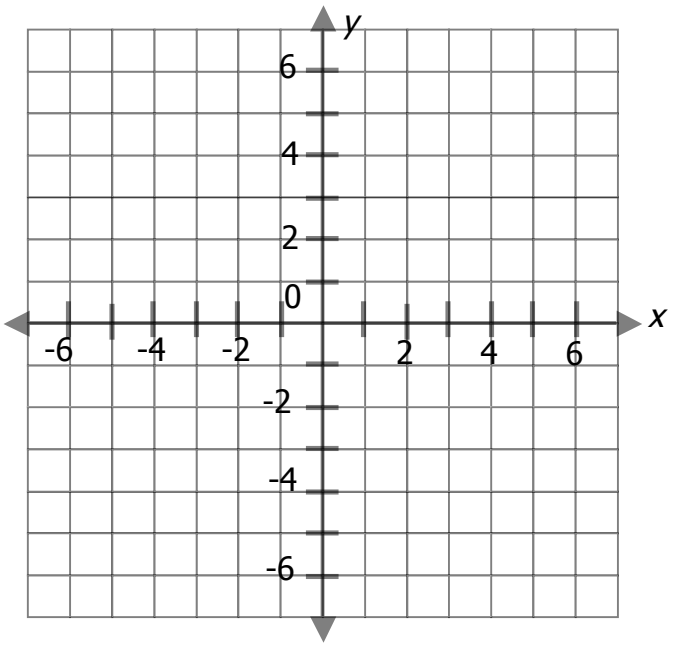
a.) Write an inequality statement to represent the heights of normal riding horses *in inches.*

b.) Write an inequality statement stating the heights, in inches, of horses that do not fit the normal riding horse height specifications.

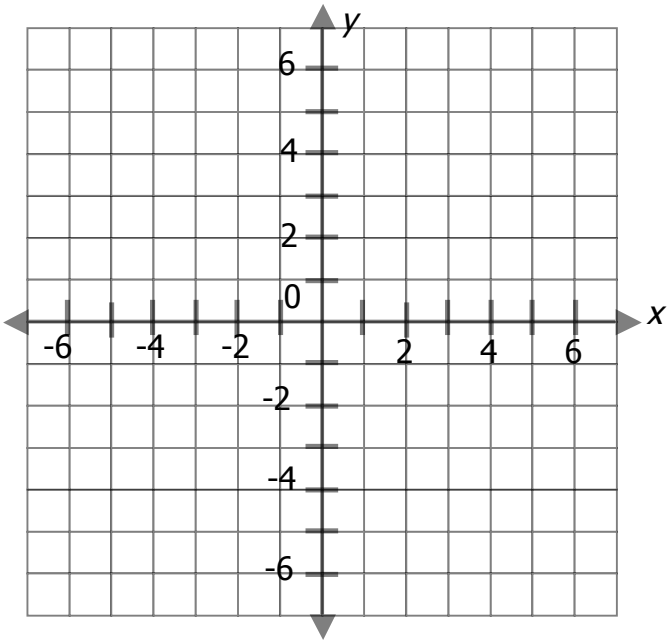
PS #5.3 – Two-Variable Inequalities

For 27 – 32, graph the following inequalities.

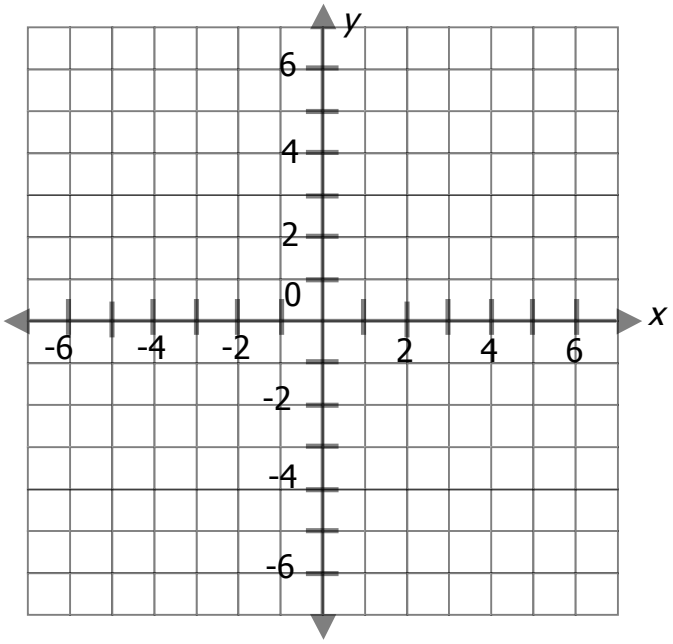
27.) $y \geq \frac{1}{2}x + 1$



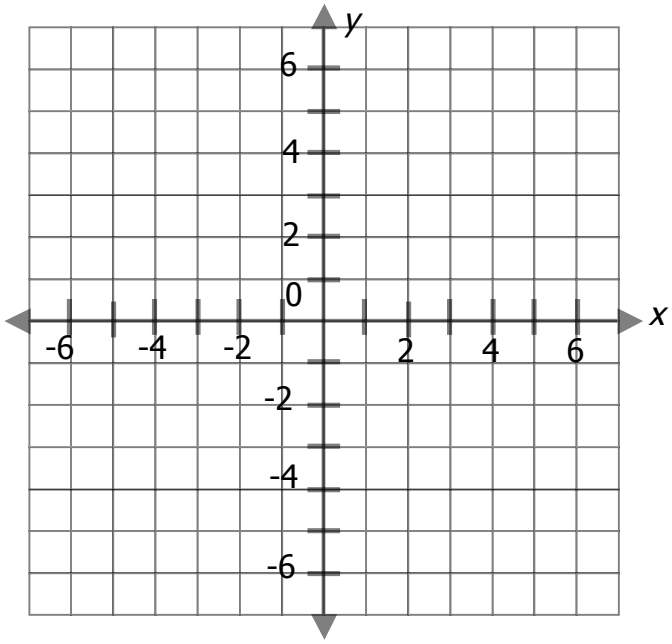
28.) $y < 3$



29.) $2x - 3y \geq 9$

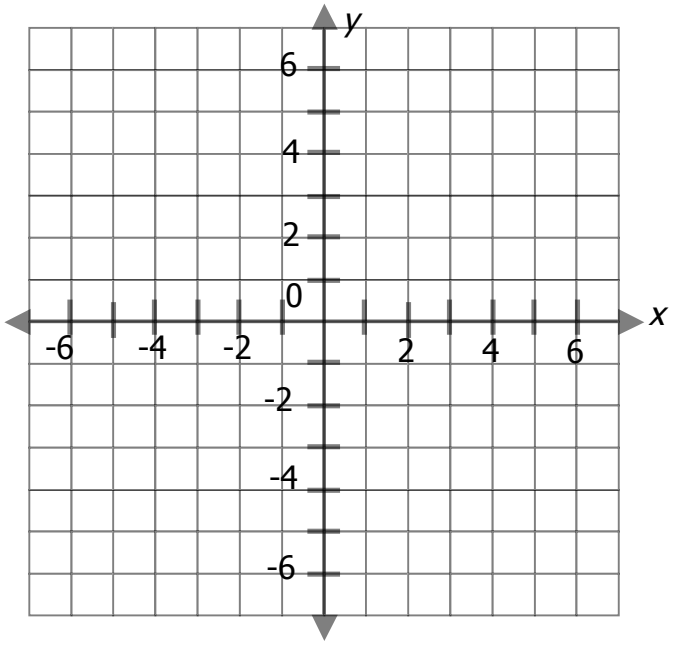


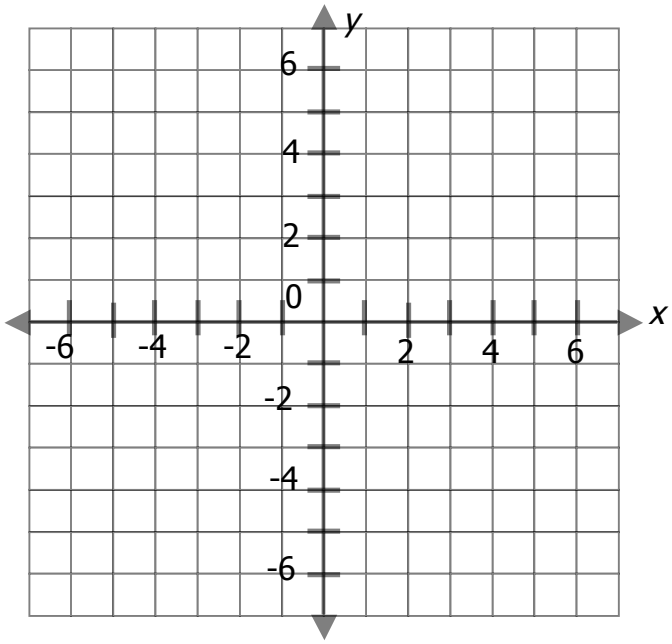
30.) $2x + 3y < 9$



31.) $x \geq 3$

32.) $2x + 5y < 15$





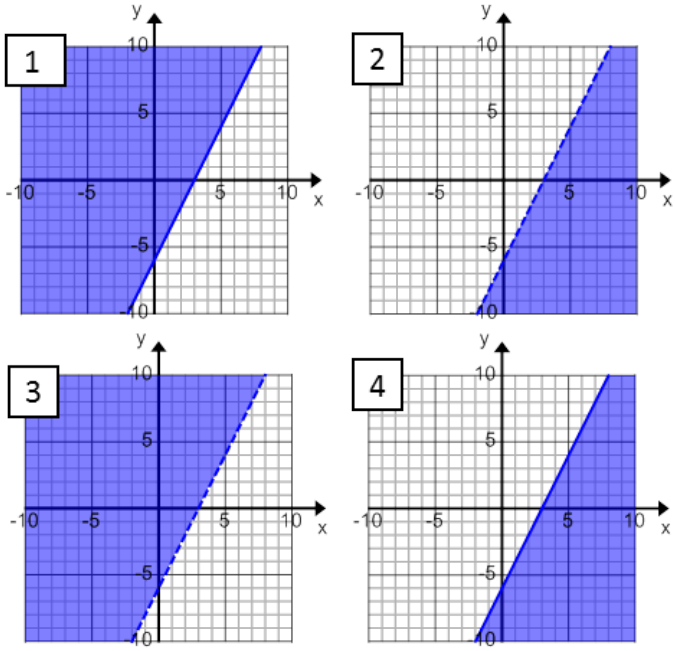
33.) Match each inequality with its graph. Explain your reasoning.

a.) $2x - y > 6$

b.) $y \leq 2x - 6$

c.) $2x < y + 6$

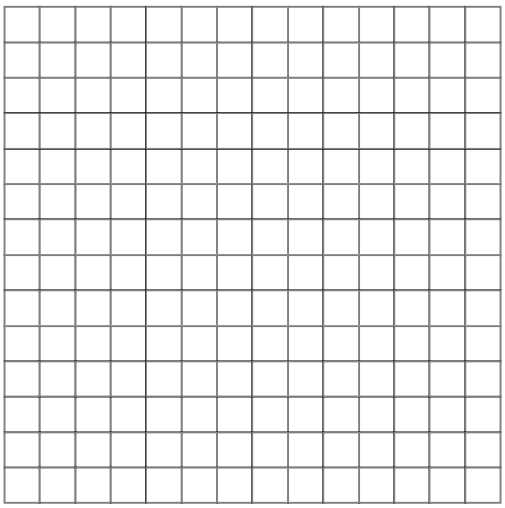
d.) $2x - 6 \leq y$



34.) Marti sells tacos and burritos from a food truck at the farmers market. She sells burritos for $3.50 each, and tacos for $2.00 each. She hopes to earn at least $120 at the farmers market this Saturday.

a.) Create a linear inequality that represents the solution to this problem. Let x equal the number of burritos Marti sells and let y equal the number of tacos she sells.

b.) Graph your inequality in the coordinate plane.



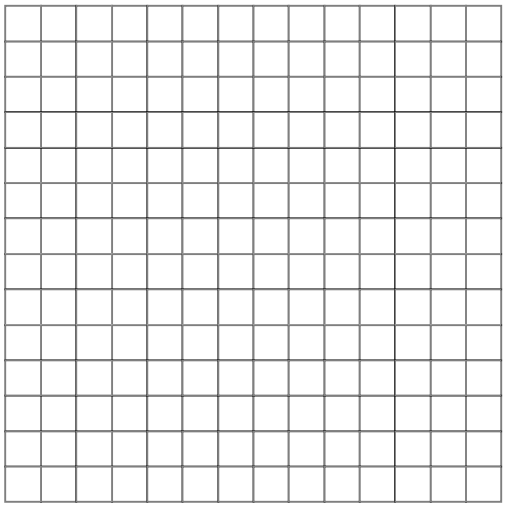
c.) Identify 3 combinations of tacos and burritos that will earn Marti more than $120.

d.) Identify 3 combinations of tacos and burritos that will earn Marti exactly $120.

e.) Identify 3 combinations of tacos and burritos that will *not* earn Marti at least $120.

f.) Are the points (10,49.5) a solution to the inequality you created? Explain your answer.

35.) Suppose you intend to spend no more than $60 buying books. Hardcover books cost $12 and paperback books cost $5. How many of each type of book can you buy?

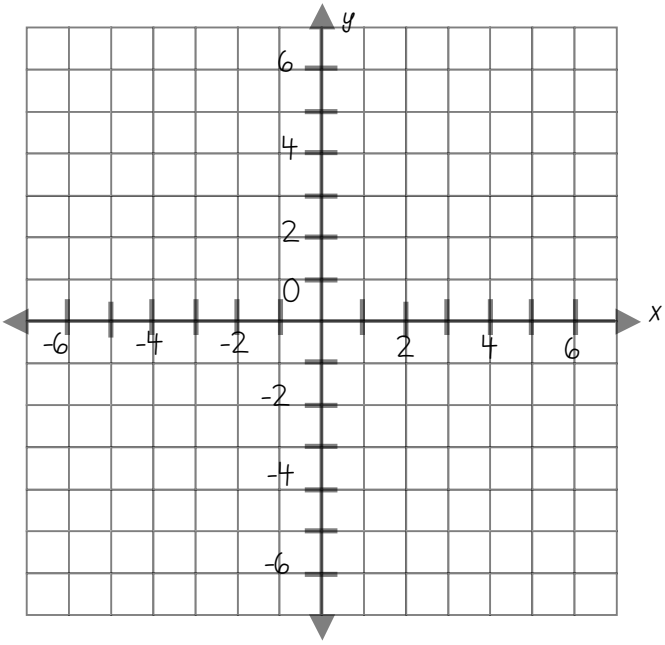


36.) Marissa tells her friend that she is holding 20 coins all of which are dimes and quarters. They have a value of $4.10. She says she will give him the coins if he can tell her how many of each she is holding. Solve this problem for her friend.

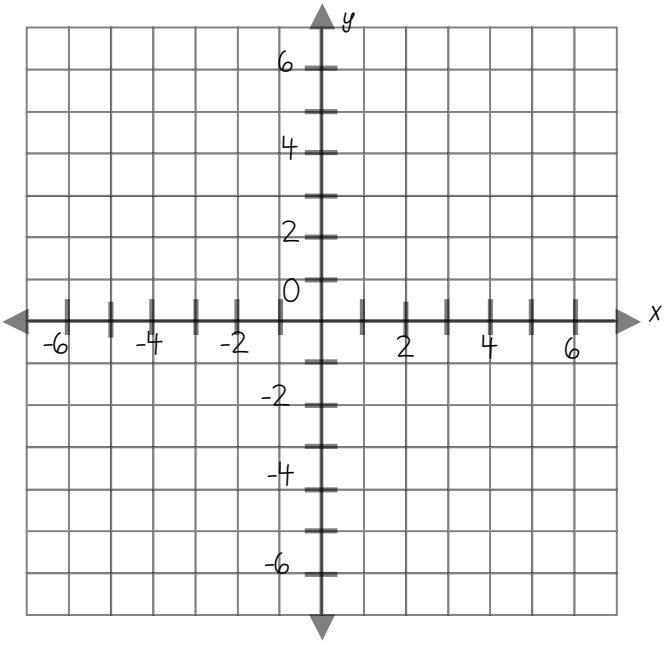
PS #5.4 – Systems of Inequalities

For 37 – 40, solve the system of linear equations/inequalities. If it is a system of linear equations, state the solution. If it is a system of linear inequalities, state the solution. If it is a system of linear inequalities, state one point that satisfies the solution set.

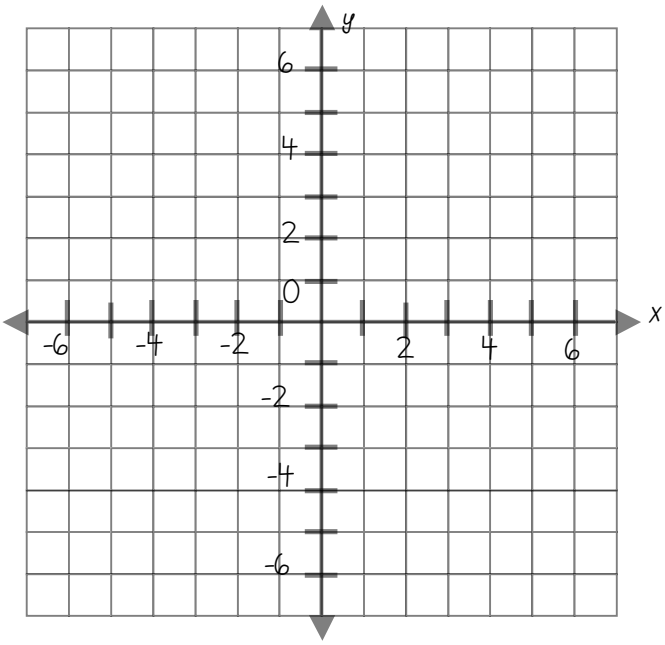
37.) $y = x + 1$
$y - 3x = -7$



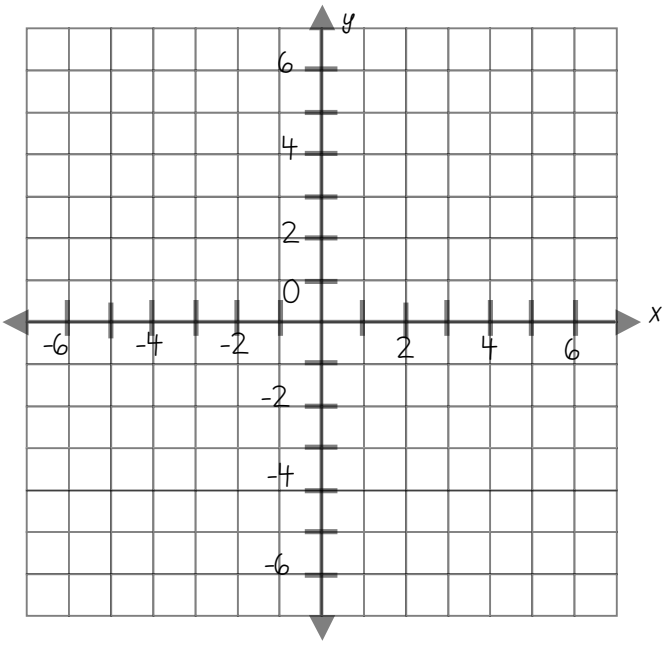
38.) $x > 3$
$y < 2$



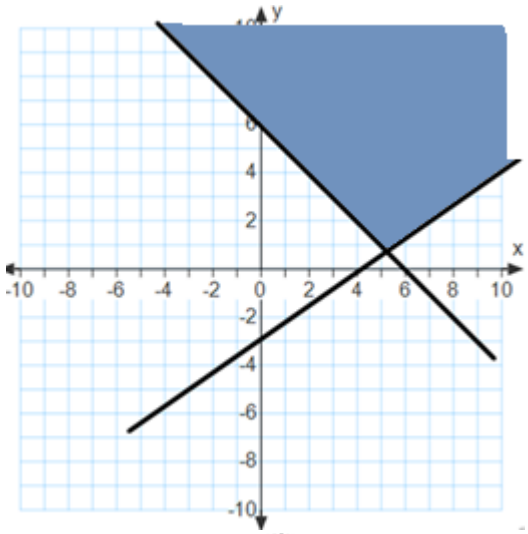
39.) $y \leq x + 2$
$y \geq x - 2$



40.) $2x + y < 0$
$2y - 6 \geq 5x$



41.) Write a system of inequalities that represents the shaded region on the following graph.

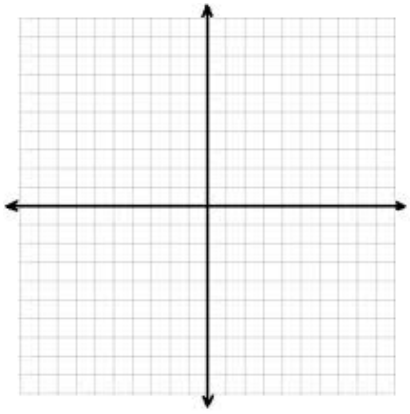


42.) Graph the solution to the following system of inequalities.

$x \geq 0$

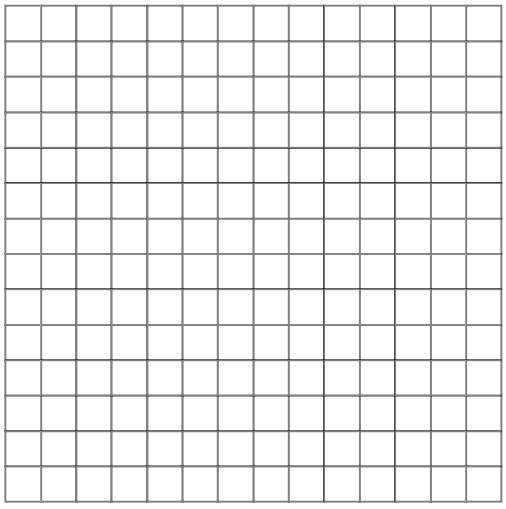
$y < 2$

$x + 3y > 0$



43.) Miss Jones wants to buy at least 10 books. Each paperback book costs an average of $10 and each hardcover book costs an average of $20. Miss Jones is planning to spend less than $240 on books.

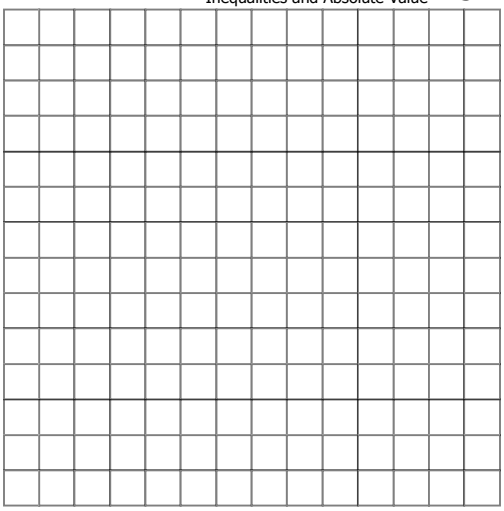
a.) Write and graph a system of inequalities to model the number of books she can purchase to stay under budget. Be sure to label all parts of your graph.



b.) State one solution that would work. How much did she spend on hardcover books?

44.) The science club wants to buy t-shirts for the organization. They have a budget of $120. Plain white t-shirts cost $3 and red t-shirts with writing costs $6. There are 20 students in the club but all are not sure if they are going to purchase a shirt.

a.) Write and graph a system of inequalities to model the number of t-shirts the club can purchase to stay under budget.



b.) State one solution that would work. How many more white t-shirts could they buy? Explain.

45.) It took 3 hours for a plane, flying against the wind, to travel 900 miles from Alabama to Minnesota. The "ground speed" of the plane is 300 miles per hour. On the return trip, the flight took only 2 hours with a ground speed of 450 miles per hour. During both flights the speed and the direction of the wind were the same. Find the speed of the plane in still air and the speed of the wind.

PS #5.5 – Absolute Value Equations

Solve each equation.

46.) $\lvert n-3\rvert = 5$	47.) $\lvert 4x-8\rvert = 20$
48.) $\lvert 6y-7\rvert = -1$	49.) $\left\lvert \frac{3}{4}a-3\right\rvert = 9$
50.) $\lvert 3x-1\rvert + 5 = 3$	51.) $5\lvert c-2\rvert = 30$
52.) $\lvert x-2\rvert = 2x-3$	53.) $\lvert 3x+2\rvert = 4x+5$

54.) $\lvert 2a+1\rvert = a+5$	55.) $\frac{4}{\lvert p\rvert}+12=14$

Write an equation involving absolute value for each set of solutions.

56.) $\{-4,4\}$	57.) $\{-3,5\}$
58.) $\{-6,2\}$	59.) $\left\{-\frac{4}{3},\frac{2}{3}\right\}$

60.) To allow for a model's height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of skirts is 20 inches.

a.) Write an absolute value equation that represents the length of the skirts.

b.) What is the range of the lengths of the skirts?

c.) If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot tall model?

61.) On the starship Enterprise, Captain Picard continually asks the food replicator to brew him a cup of Earl Grey tea. The proper brewing temperature for Earl Grey tea is 210°F plus or minus 5 degrees. Write and solve an absolute value equation representing the maximum and minimum brewing temperatures for Earl Grey tea.

62.) Explain why there are either two, one, or no solutions for absolute value equations. Demonstrate an example of each possibility.

63.) Determine whether the following statements are *sometimes, always,* or *never* true, if c is an integer. Explain your reasoning.

a.) The value of $|x+1|$ is greater than zero.

b.) The solution of $|x+c|=0$ is greater than 0.

c.) The equality $|x|+c=0$ has no solution.

d.) The value of $|x+c|+c$ is greater than 0.

PS #5.6 – Absolute Value Inequalities

Solve each equation. Then graph the solution set.

64.) $\lvert 2x \rvert \geq 6$	65.) $\lvert 2x - 3 \rvert > 5$
66.) $\lvert 2x + 3 \rvert < x + 6$	67.) $\lvert x + 2 \rvert > 2$
68.) $\lvert 5 - 4x \rvert + 2 < 6$	69.) $\lvert 2x + 5 \rvert + 4 \geq 1$

70.) $\left\|\frac{x}{2}+6\right\| \le 10$	71.) $6 < \|x+1\| < 8$

72.) Find the absolute-value inequality statement that corresponds to the inequality $-2 < x < 4$.

73.) Find the absolute-value inequality statement that corresponds to the inequality $x \le 19$ or $x \ge 24$.

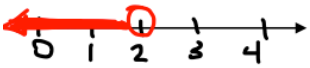
74.) Forensic scientists use the equation $h = 2.6f + 47.2$ to estimate the height, h, of a woman given the length in centimeters, f, of her femur bone. Suppose the equation has a margin of error of ± 4 centimeters and the length of a female skeleton's femur is 48 centimeters.

a.) Write and solve an absolute value inequality that describes the woman's height in centimeters.

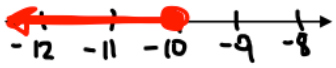
b.) Convert your answer to inches (1 in = 2.54 cm). Round the values to the nearest hundredth.

Selected Answers:

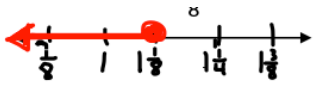
1.) $y < 2$, $(2,\infty)$



2.) $n \le -10$, $(-\infty,-10]$



3.) $x \le 1.125$ or $x \le \frac{9}{8}$, $[1.125,\infty)$



4.) $m \ge 1.5$ or $m \ge \frac{3}{2}$, $[1.5,\infty)$

5.) $25

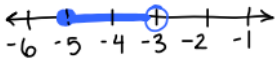
6.) length = 65 cm

7.) Sue: 39 lb
Karen: 117 lb

8.) $x < 35$

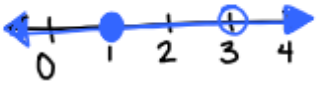
9.) 13

10.) [-5,3)

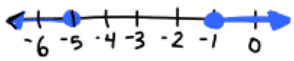


11.) no solution

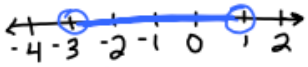
12.) $(-\infty,\infty)$



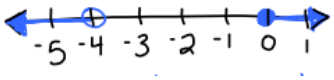
13.) $(-\infty,-5]\cup[-1,\infty)$



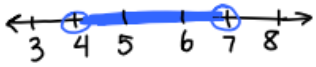
14.) (-3,1)



15.) $(-\infty,-4]\cup[0,\infty)$



16.) (4,7)

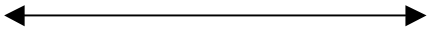


17.) [-3,2)

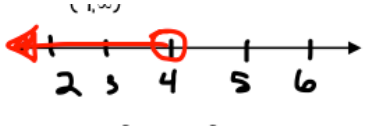
18.) $-3 < x < 1$

19.) $x < -2$ or $x \ge 0$

20.) $n > 1$ or $n \le 0$
$(-\infty,0]\cup(1,\infty)$

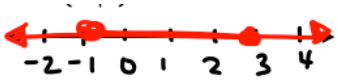


21.) $w < 4$
$(4,\infty)$

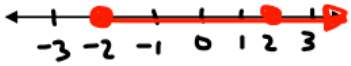


22.) $3 < x \le 6$
$(3,6]$

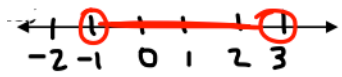
23.) $x \ge -1$ or $x \le 3$
$(-\infty,\infty)$



24.) $z \ge -2$ or $z \ge 2$
$[-2,\infty)$

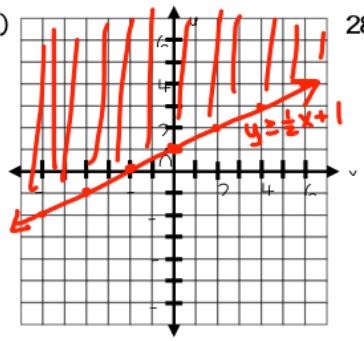


25.) $-1 < y < 3$

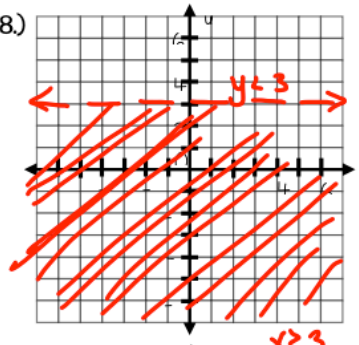


(-1,3)

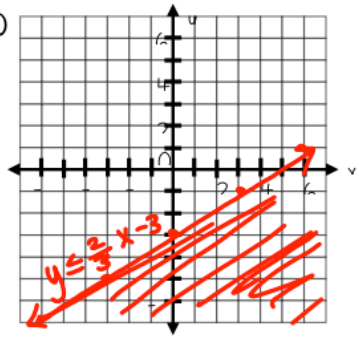
27.)



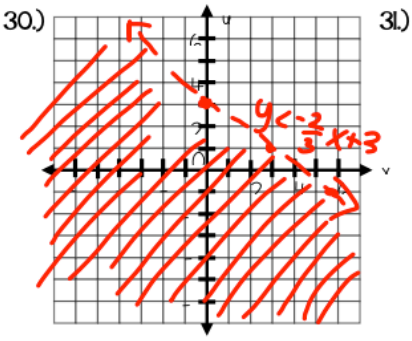
28.)



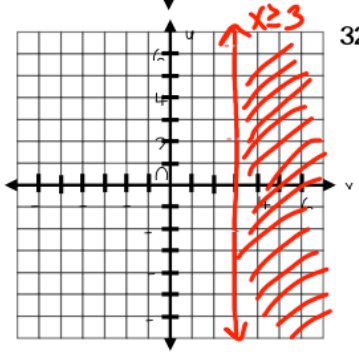
29.)



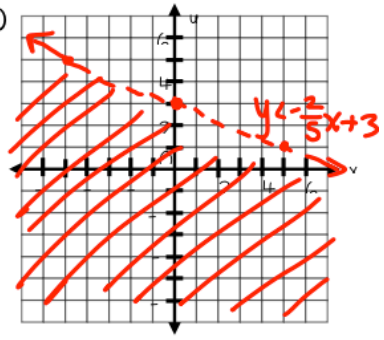
30.)



31.)



32.)



33.) We will discuss in class.

34.) a.) Let x = # of tacos

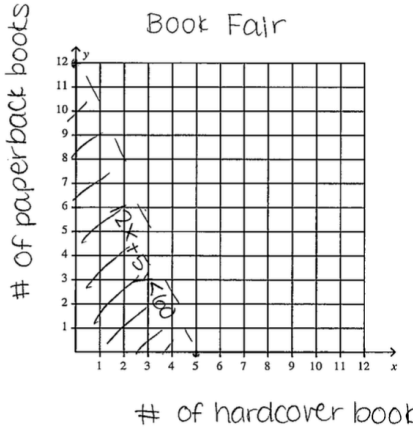
Let y = # of burritos

$3.5x + 2y \geq 120$

f.) yes

35.) Let x = # of hardcover books

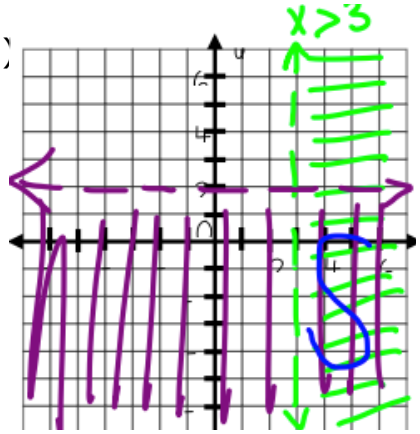
Let y = # of paperback books



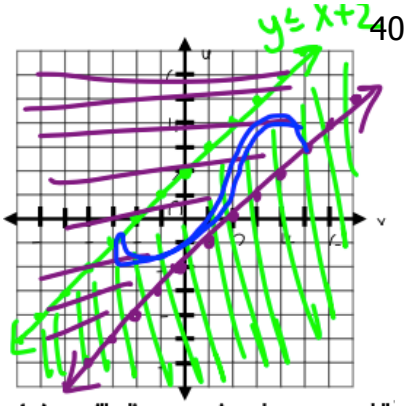
36.) 14 quarters, 6 dimes

37.) (4,5)

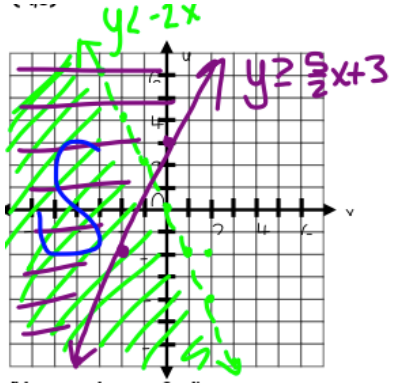
38.)



39.)



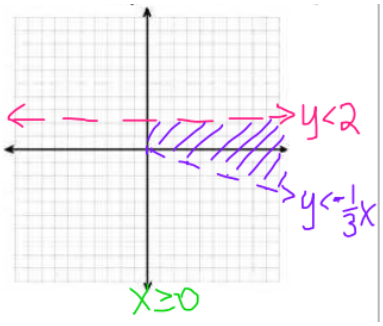
40.)



41.) $y \geq \frac{2}{3}x - 3$

$y \leq -x + 6$

42.)



43.) Let x = # of paperback books

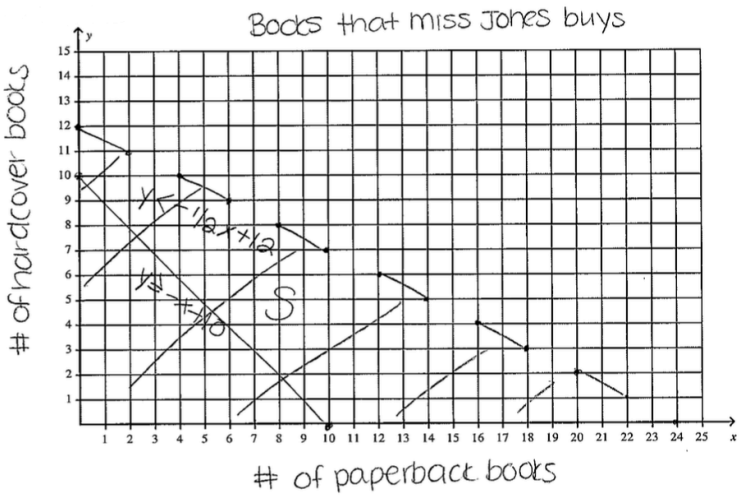
Let y = # of hardcover books

10x + 20y ≤ 240

x + y ≥ 10

(x and y are whole #s)

b.) Answers vary



44.) Let x = # of white t's

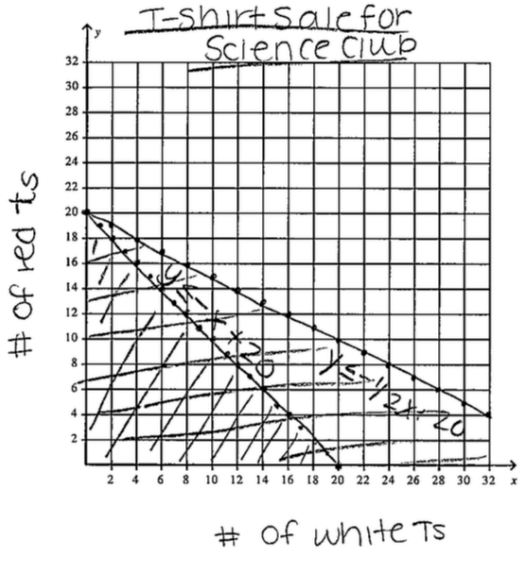
Let y = # of red t's

3x + 6y ≤ 120

x + y ≤ 20

(x and y are whole #s)

b.) Answers vary



45.) Plane: 375 mph

Wind: 75 mph

46.) $\{-2,8\}$

47.) $\{-3,7\}$

48.) no solution

49.) $\{-8,16\}$

50.) no solution

51.) $\{-4,8\}$

52.) $\frac{5}{3}$

53.) -1

54.) $\{4,-2\}$

55.) $\{\pm 2\}$

56.) $|x| = 4$

57.) $|x-1| = 4$

58.) $|x+2| = 4$

59.) $\left|x+\frac{1}{3}\right| = 1$

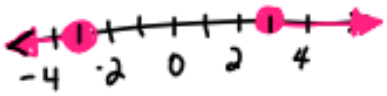
60.) b.) $18 \le x \le 22$ c.) No

61.) $205^\circ F \le x \le 215^\circ$

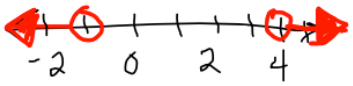
62.) We will discuss in class

63.) We will discuss in class

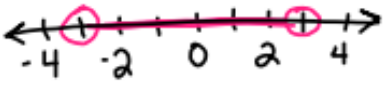
64.) $x \ge 3$ or $x \le -3$, $(-\infty,-3]\cup[3,\infty)$



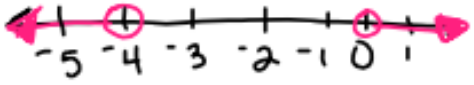
65.) $x < -1$ or $x > 4$, $(-\infty,-1)\cup(4,\infty)$



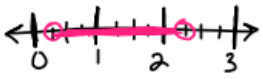
66.) $-3 < x < 3$, $(-3,3)$



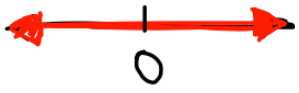
67.) $x > 0$ or $x < -4$, $(-\infty,-4)\cup(0,\infty)$



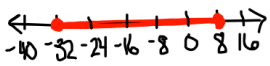
68.) $\frac{1}{4} < x < \frac{9}{4}$, $\left(\frac{1}{4},\frac{9}{4}\right)$



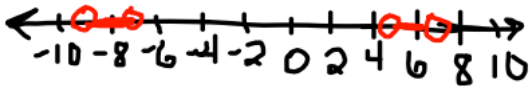
69.) all reals, $(-\infty,\infty)$



70.) $-32 \le x \le 8$, $[-32,8]$



71.) $-9 < x < -7$ or $5 < x < 7$, $(-9,-7)\cup(5,7)$



72 – 74, we will discuss in class