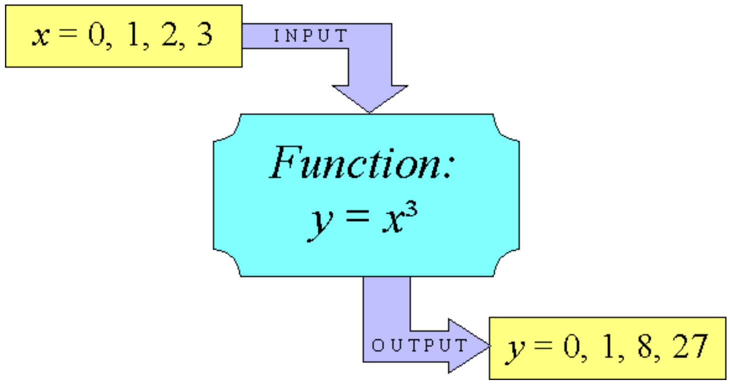
Unit 8 Notes
Functions



Tentative Schedule

Day	Classwork	Assignment
Mon. 1/26 Tues. 1/27	Test #7	Video #8.1 with Notes: Introduction to Functions and Function Notation
Wed. 1/28	1 – 11	Video #8.2 with Notes: Graphs of Functions and their Features
Thurs. 1/29 Fri. 1/30	12 – 18	Video #8.3 with Notes: Average Rate of Change
Mon. 2/2	19 – 26	Video #8.4 with Notes: Domain and Range
Tues. 2/3 Wed. 2/4	27 – 30	Finish Problem Set Optional Review Sheet
Thurs. 2/5	**Quest #8**	Video #9.1 with Notes

Name: ____

Notes 8.1 - Introduction to Functions and Function Notation

The concept of the **function** ranks near the top of the list in terms of important Algebra concepts. Almost all of higher-level mathematical modeling is based on the concept. Like most important ideas in math, it is relatively simple:

THE DEFINITION OF A FUNCTION

A **function** is a clearly defined **rule** that converts an **input** into **at most one output**. These rules often come in the form of: (1) equations, (2) graphs, (3) tables, and (4) verbal descriptions.

Exercise #1: Consider the function rule: multiply the input by two and then subtract one to get the output.

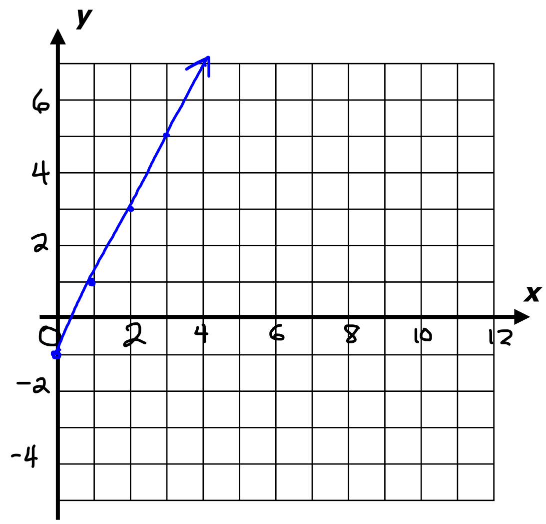
a.) Fill in the table below for inputs and outputs. Inputs are often designated by *x* and outputs by *y*.

Input x	Calculation	Output y
0	$2(0)-1=0-1=-1$	-1
1	$2(1)-1=2-1=1$	1
2	$2(2)-1=4-1=3$	3
3	$2(3)-1=6-1=5$	5

(b) Write an equation that gives this rule in symbolic form.

$y=2x-1$

(c) Graph the function rule on the graph paper shown below. Use your table in (a) to help.



Exercise #2: In the function rule from #1, what input would be needed to produce an output of 17? Why is it harder to find an input when you have an output than finding an output when you have an input? $y=17$

$y=2x-1$
$17=2x-1$
$+1 \quad +1$
$18=2x$
$\boxed{9=x}$

It's harder b/c essentially you have to work backwards.

Exercise #3: A function rule takes an input, *n*, and converts it into an output, *y*, by increasing one half of the input by 10. Determine the output for this rule when the input is 50 and then write an equation for the rule.

$\boxed{y=\frac{1}{2}n+10}$

$y=\frac{1}{2}(50)+10$

$y=30+10$

$y=\boxed{40}$

Since functions are rules that convert **inputs** (typically *x*-values) into **outputs** (typically *y*-values), it makes sense that they must have their own **notation** to indicate what the what the rule is, what the input is, and what the output is. In the first exercise, your teacher will explain how to interpret this notation.

Exercise #4: For each of the following functions, find the outputs for the given inputs.

(a) $f(x)=3x+7$

$f(2)=3(2)+7$
$=6+7=\boxed{13}$

$f(-3)=3(-3)+7$
$=-9+7=\boxed{-2}$

(b) $g(x)=\dfrac{x-6}{2}$

$g(20)=\dfrac{20-6}{2}=\dfrac{14}{2}=\boxed{7}$

$g(0)=\dfrac{0-6}{2}=\dfrac{-6}{2}=\boxed{-3}$

(c) $h(x)=\sqrt{2x+1}$

$h(4)=\sqrt{2(4)+1}=\sqrt{9}=\boxed{3}$

$h(0)=\sqrt{2(0)+1}=\sqrt{1}=\boxed{1}$

Function notation can be very, very confusing because it really looks like multiplication due to the parentheses. But, there is no multiplication involved. The notation serves two purposes: (1) to tell us what the rule is and (2) to specify an output for a given input.

FUNCTION NOTATION

$$y = f\ (x)$$

Output ⟶ y Rule ⟶ f Input ⟶ x

Exercise #5: Given the function $f(x)=\dfrac{x}{3}+7$ do the following.

(a) Explain what the function rule does to convert the input into an output.

Take the input divide it by 3, then increase by 7 to get an output

(b) Evaluate $f(6)$ and $f(-9)$.

$f(6)=\dfrac{6}{3}+7=2+7=\boxed{9}$

$f(-9)=\dfrac{-9}{3}+7=-3+7=\boxed{4}$

(c) Find the input for which $f(x)=13$. Show the work that leads to your answer.

$f(x)=17$ $\quad f(x)=\dfrac{x}{3}+7$

$3\cdot 17=\dfrac{x}{\cancel{3}}\cdot\cancel{3}+7\cdot 3$

$51=x+21$
$-21 \quad -21$

$\boxed{30=x}$

(d) If $g(x)=2f(x)-1$ then what is $g(6)$? Show the work that leads to your answer.

$f(x)=\dfrac{x}{3}+7$

$g(x)=2\left(\dfrac{x}{3}+7\right)-1$

$g(6)=2\left(\dfrac{6}{3}+7\right)-1$

$g(6)=2(2+7)-1$

$g(6)=2(9)-1$

$g(6)=18-1=\boxed{17}$

Recall that function rules commonly come in one of three forms: (1) equations (as in Exercise #1), (2) graphs, and (3) tables. The next few exercises will illustrate function notation with these three forms.

***Exercise #6*:** Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, *T*, is a function of the number of hours, *h*.

h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ $(^\circ F)$	212	141	104	85	76	70	68	66	65

(a) Evaluate $T(2)$ and $T(6)$.

$T(2)=\boxed{104^\circ}$ $T(6)=\boxed{68^\circ}$

(b) For what value of *h* is $T(h)=76$?

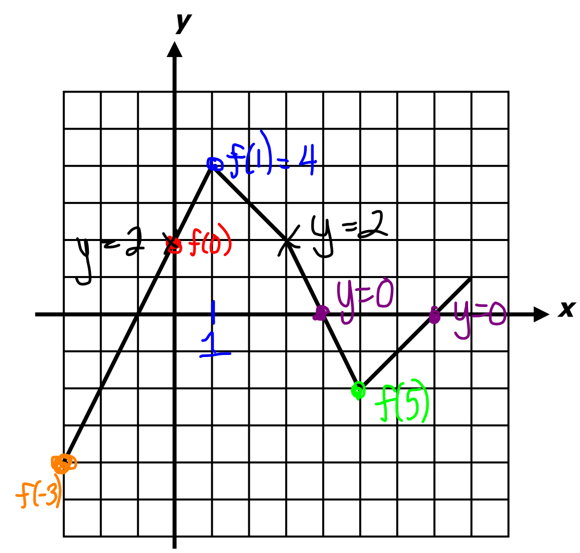
$T(4)=76$ $\boxed{h=4}$

(c) Between what two consecutive hours will $T(h)=100$? Explain how you arrived at your answer.

Between hours 2 and 3.

The temperature keeps decreasing and @ h=2 it was above 100° and @ h=3 it's less than 100°.

***Exercise #7*:** The function $y=f(x)$ is defined by the graph shown below. It is known as **piecewise linear function** because it is made up of **straight line segments**. Answer the following questions based on this graph.



(a) Evaluate each of the following:

$f(1)=$ 4
x=1

$f(5)=$ −2

$f(-3)=$ −4

$f(0)=$ 2

(b) Solve each of the following for all values of the input, *x*, that make them true.

$f(x)=0$ $\boxed{x=4 \quad x=7}$
y=0

$f(x)=2$ $\boxed{x=0 \quad x=3}$
y=2

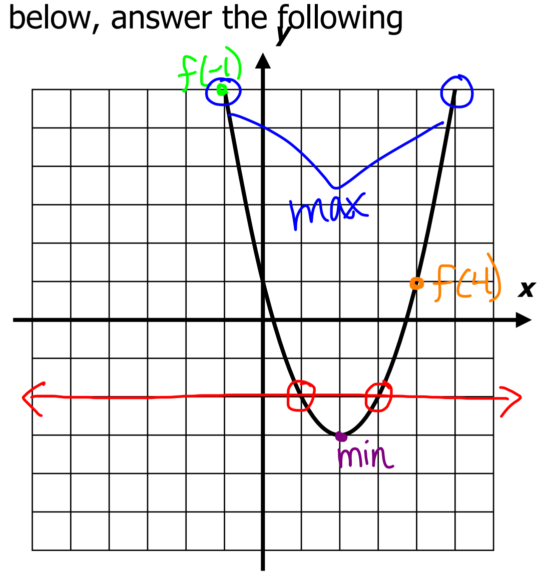
(c) What is the largest output achieved by the function? At what *x*-value does it hit?

Look for the highest point
(y=4)
It happens when $\boxed{x=1}$

Notes 8.2 - Graphs of Functions and Their Features

Graphs are one of the most powerful ways of visualizing a function's rule because you can quickly read **outputs** given **inputs**. You can also easily see features such as **maximum and minimum** output values. Let's review some of those skills in Exercise #1.

Exercise #1: Given the function $y = f(x)$ defined by the graph below, answer the following questions.



(a) Find the value of each of the following:

$f(4) =$ 1 $\qquad$ $f(-1) =$ 6

(b) For what values of *x* does $f(x) = -2$? Illustrate on the graph.

y=-2

$\boxed{\begin{array}{l} X=1 \\ X=3 \end{array}}$

(c) State the **minimum** and **maximum values of** the **function**.

min=-3 $\qquad$ max=6

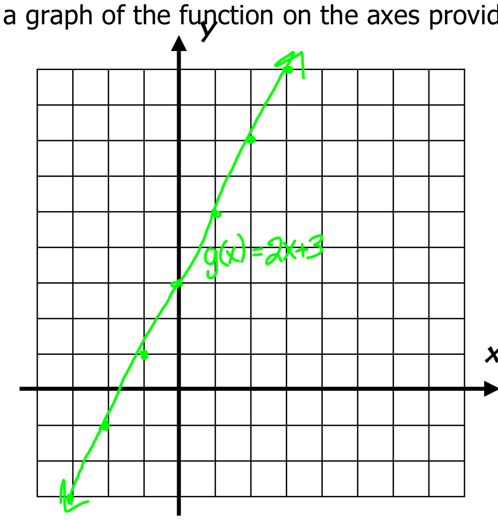
So, if we can read a graph to produce outputs (*y*-values) if we are given inputs (*x*-values), then we should be able to reverse the process and produce a graph of the function from its **algebraically expressed rule.**

Exercise #2: Consider the function given by the rule $g(x) = 2x + 3$.

(a) Fill out the table below for the inputs given.

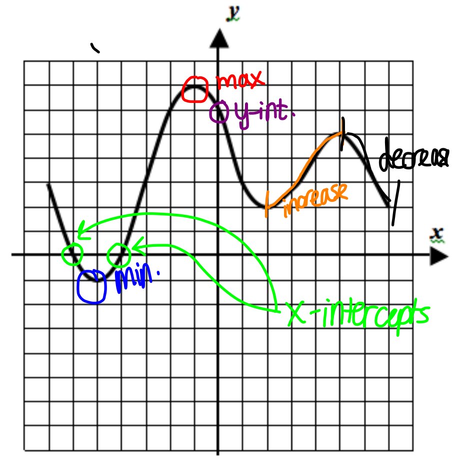
x	$2x+3$	(x, y)
-3	2(-3)+3 = -3	(-3,-3)
-2	2(-2)+3 = -1	(-2,-1)
-1	2(-1)+3 = 1	(-1,1)
0	2(0)+3 = 3	(0,3)
1	2(1)+3 = 5	(1,5)
2	2(2)+3 = 7	(2,7)
3	2(3)+3 = 9	(3,9)

(b) Draw a graph of the function on the axes provided.



There is a lot of terminology associated with the **graph of a function**. Many of the terms have names that are descriptive, but still, work is needed to master the ideas.

Exercise #3: The function $y = f(x)$ is shown graphed below over the interval $-7 \le x \le 7$.



(a) Find the maximum and minimum values of the function. State the values of *x* where they occur as well.

min=-1, occurs @ x = -5.

min=7, occurs @ x=-1

(b) What is the *y*-intercept of the function? Explain why a function cannot have more than one *y*-intercept.

y-int: (0,6)

A function can only have 1 y-int. b/c y-int. only occurs when x=0. Each input can only have one output.

(c) Give the *x*-intercepts of the function. These are also known as the function's **zeroes** because they are where $f(x) = 0$.

x=-6
or
x=-4

(d) Would you characterize the function as **increasing or decreasing** on the domain interval $-5 \le x \le -1$? Explain your choice.

The function is increasing because as x increases, so does f(x).

(e) one additional interval over which the function is increasing and one over which it is decreasing.

Increasing: (2,4) or 2<x<4

Decreasing: (4,6) or 4<x<6

(f) The following points are known as **turning points**. Each can be classified as a **relative maximum** or a **relative minimum**. State which you think each is.

$(-5,-1)$	$(-1,7)$	$(2,2)$	$(5,5)$
relative minimum or relative maximum	relative minimum or relative maximum	relative minimum or relative maximum	relative minimum or relative maximum

Sometimes the function's rule gets all sorts of funny and can include being **piecewise defined**. These functions have different rules for different values of *x*. These separate rules combine to make a larger (and more complicated rule). Let's try to get a feel for one of these and mix in our **function terminology** while we are at it.

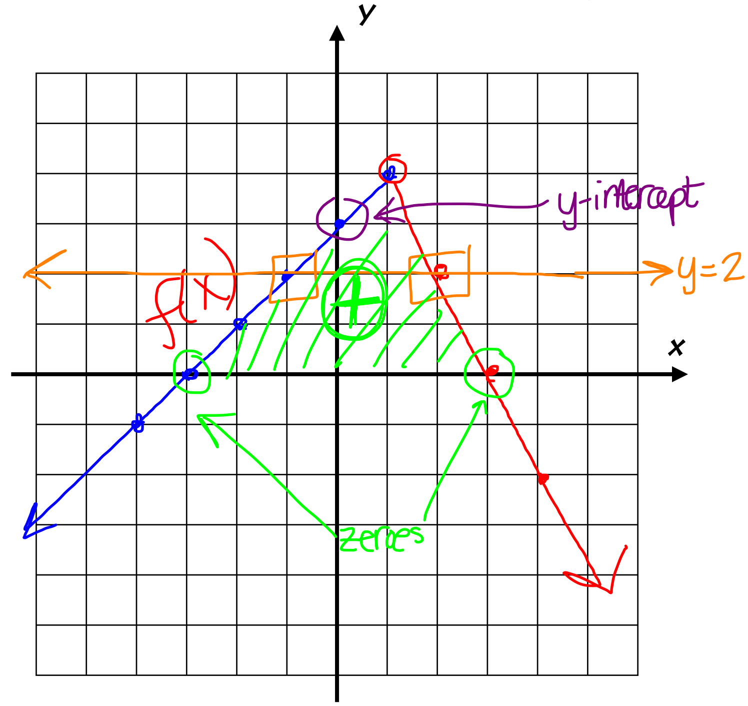
Exercise #4: Consider the **piecewise linear** function given the equation $f(x)=\begin{cases} x+3 & x\le 1 \\ 6-2x & x>1 \end{cases}$.

(a) Create a table of values for this function below over the interval $-4\le x\le 4$. Then create a graph on the axes for this function.

x+3

6-2x

x	Rule/Calculation	(x, y)
-4	$-4+3=-1$	$(-4,-1)$
-3	$-3+3=0$	$(-3,0)$
-2	$-2+3=1$	$(-2,1)$
-1	$-1+3=2$	$(-1,2)$
0	$0+3=3$	$(0,3)$
1	$1+3=4$	$(1,4)$
2	$6-2(2)=2$	$(2,2)$
3	$6-2(3)=0$	$(3,0)$
4	$6-2(4)=-2$	$(4,-2)$



(b) State the **zeroes of the function**.

$X=-3$
$X=3$

(c) State the function's *y*-intercept.

$(0,3)$

(d) Give the interval over which the function is increasing. Give the interval over which it is decreasing.

Increasing: $(-\infty,1)$ or $x<1$

Decreasing: $(1,\infty)$ or $x>1$

(e) Give the coordinates of the one turning point and classify it as either a relative maximum or relative minimum.

$(1,4)$ – relative max

(f) Use your graph to find all solutions to the equation $f(x)=2$. Illustrate your solution graphically and find evidence in the table you created.

$y=2$

$\boxed{X=-1 \text{ or } X=2}$

(g) State the interval over which this function is positive. How can you tell this quickly from the graph?

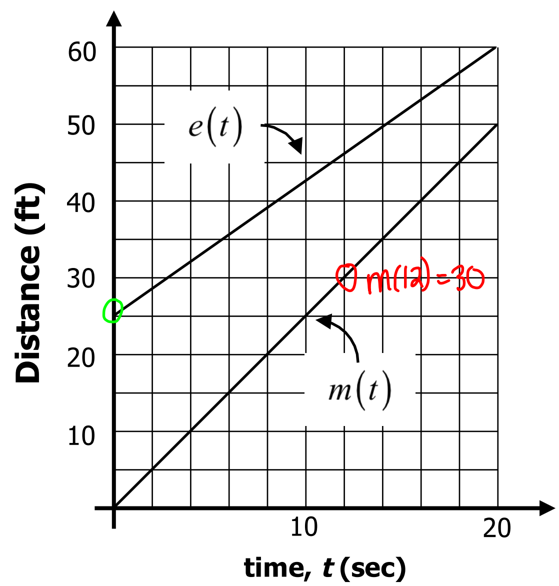
$(-3,3)$ or $-3<X<3$

Above the x-axis.

Notes 8.3 - Average Rate of Change

Functions are rules that give us **outputs** when we supply them with **inputs**. Very often, we want to know how **fast** the outputs are changing compared to a change in the input values. This is referred to as the **average rate of change** of a function.

Exercise #1: Max and his younger sister Evie are having a race in the backyard. Max gives his sister a head start and they run for 20 seconds. The distance they are along in the race, in feet, is given below with Max's distance given by the function $m(t)$ and Evie's distance given by the function $e(t)$.



(a) How do you interpret the fact that $m(12)=30$? Illustrate your response by using the graph.

After 12 sec, Max has gone 30 ft.

(b) If both runners start at $t=0$, how much of a head start does Max give his little sister? How can you tell?

25 ft - It's Evie's y-intercept

(c) Does Max catch up to his sister? How can you tell?

Yes, he can because he's traveling at a faster rate

(d) How far does Max run during the 20 second race? How far does Evie run? What calculation can you do to find Evie's distance?

$m(20) = 50$ ft

$e(20) - e(0) = 60 - 25 = 35$ ft.

Max runs 50ft and Evie runs 35ft.

(e) How fast do both Evie and Max travel? In other words, how many feet do each of them run per second? Express your answers as decimals and attach units.

MAX'S SPEED (FEET PER SECOND)

50ft in 20 sec

$\frac{50 \text{ ft}}{20 \text{ s}} = \boxed{2.5 \text{ ft/s}}$

EVIE'S SPEED (FEET PER SECOND)

35 ft in 20 sec

$\frac{35 \text{ ft}}{20 \text{ s}} = \boxed{1.75 \text{ ft/sec.}}$

In the first exercise we were calculating the **rate** that the **function's output (y-values)** were changing compared to the **function's input (or x-values)**. This is known as finding the **average rate of change** of the function. You might think you've seen this before. And you have.

Exercise #2: Finding the average rate of change is the same as finding the Slope of a line.

There is, of course, a formula for finding average rate of change. Let's get it out of the way.

AVERAGE RATE OF CHANGE

For the function $y = f(x)$, the average rate that $f(x)$ changes from $x = a$ to $x = b$ is given by:

$$\frac{f(b)-f(a)}{b-a} = \frac{\text{how much the y-values have changed}}{\text{how much the x-values have changed}}$$

3. The table below shows the average height of a tree and the amount of years that it has been growing.

What is the average rate of change in height of the tree from year 1 to Year 5?

(A) 1 foot per year

(B) 1.25 feet per year

$\boxed{\text{(C) 1.5 feet per year}}$

(D) 2.0 feet per year

Time (years)	Height (in feet)
1	3
2	5
3	6
4	8
5	9

$f(1)=3$ $f(5)=9$

$$\frac{9-3}{5-1} = \frac{6}{4} = 1.5 \text{ ft/y}$$

4. Compute the average rate of change of f from x_1 to x_2. Round your answer to two decimal places when appropriate. Interpret your result graphically.

$f(x) = x^3 - 4x, x_1 = 2$ and $x_2 = 4$

(A) 24, the slope of the line passing through $(2,f(2))$ and $(4,f(4))$ is 24.

~~(B)~~ -24, the slope of the line passing through $(2,f(2))$ and $(4,f(4))$ is -24.

~~(C)~~ -8, the slope of the line passing through $(2,f(2))$ and $(4,f(4))$ is -8.

~~(D)~~ 8, the slope of the line passing through $(2,f(2))$ and $(4,f(4))$ is 8

① Find f(2)
$f(2)=2^3-4(2)$
$f(2)=8-8$
$f(2)=0$

② Find f(4)
$f(4)=4^3-4(4)$
$f(4)=64-16$
$f(4)=48$

③ Find
$\frac{f(4)-f(2)}{4-2} = \frac{48-0}{4-2} = \frac{48}{2} = 24$