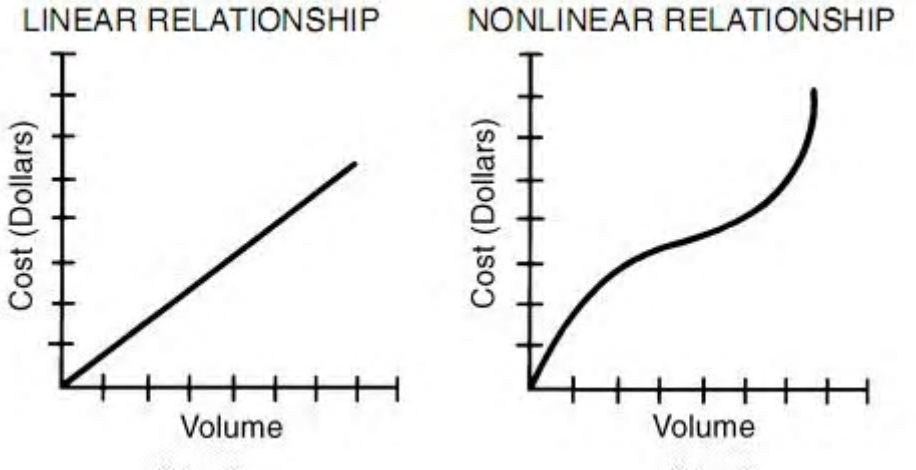
Unit 8 Notes

Linear and Non-Linear Functions



Tentative Schedule

Day	Classwork	Assignment
Mon. 3/9 Tues. 3/10	**Quiz #7**	Video #8.1 – Understanding Functions
Wed. 3/11	P.S. #8.1	Video #8.2 – Linear vs. Nonlinear Functions
Thurs. 3/12 Fri. 3/13	P.S. #8.2	Video #8.3 – Comparing Functions Day 1
Mon. 3/16	P.S. #8.3	Video #8.4 – Comparing Functions Day 2
Tues. 3/17 Wed. 3/18	P.S. #8.4	Catch-up on Checklist
Thurs. 3/19	Review for Quest #8	Review for Quest #8
Fri. 3/20 Mon. 3/23	**Quest #8**	Video #9.1 – Scatterplots

Name: _____

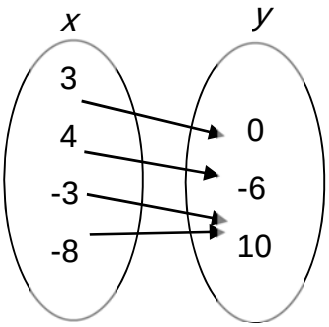
Notes 8.1 – Understanding Functions

Definition of a Function:

A relation in which each input (x-value) has only one output. (y-value)

Determine whether each relation is a function. Explain your answers.

1.) $\{(2,3),(3,0),(5,2),(-1,-2),(4,1)\}$ Yes – each input has only one output.



2.) Yes – each input has only one output

3.) $\{(2,5),(3,7),(2,1),(4,8)\}$ No – there is an input (x=2) that has more than one output (y=5 and y=1).

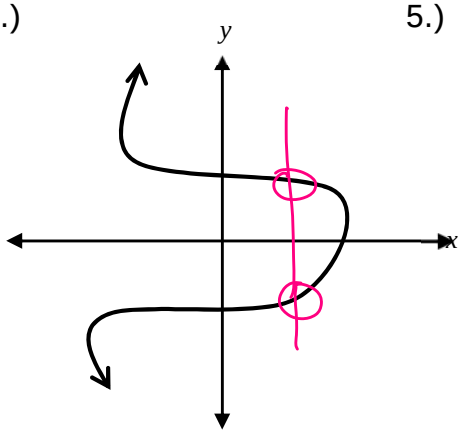
2 has more than 1 output.

Vertical Line Test

If any vertical line passes through no more than one point of the graph of a relation, then the relation is a function.

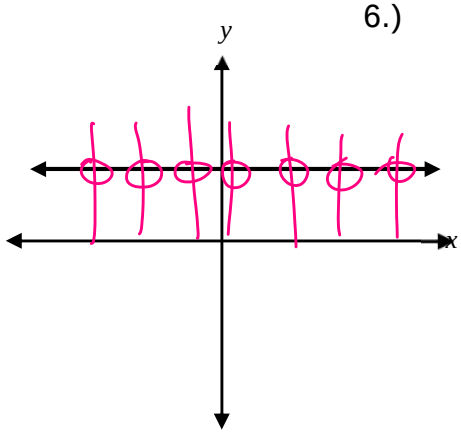
Use the vertical line test to determine if each relation is a function.

4.)



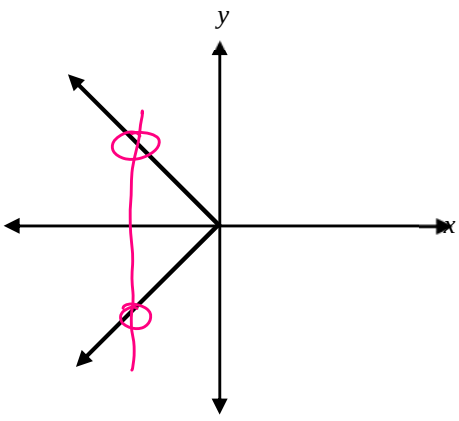
No, this fails the VLT.

5.)



Yes, this passes the VLT.

6.)

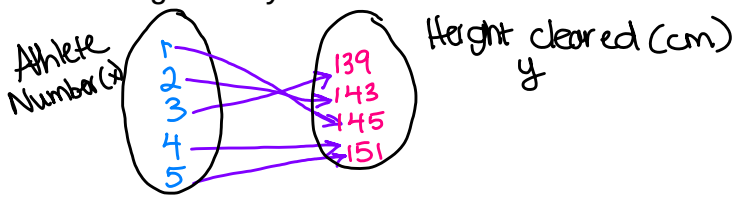


No, this Fails the VLT.

7.) The high jumpers at a track meet are wearing numbers on their uniforms. Each of the five high jumpers on the team made one jump. The height cleared by each athlete is shown in the table.

Athlete Number	1	2	3	4	5
Height Cleared (cm)	145	143	139	151	151

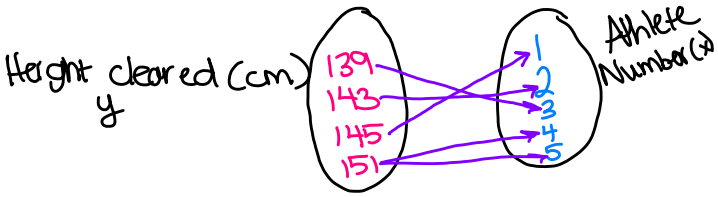
a.) Use a mapping diagram to represent the relation between the numbers of the athletes and the heights they cleared.



b.) Tell whether the relation is a function and explain why.

Yes - each input has only one output.

c.) Suppose the inputs are the heights cleared by the athletes and the outputs are the athletes' numbers. Use a mapping diagram to represent the relation. Is this relation a function?



No, there is an input that has more than one output.

8.) A tank contains 8 gallons of water. Water is then pumped into the tank at a rate of 2 gallons per minute. The total amount of water in the tank, *y*, gallons, is a function of the number of minutes, *x*, that water has been pumped into the tank.

b = 8 m = 2

a. Write an algebraic equation for the function.

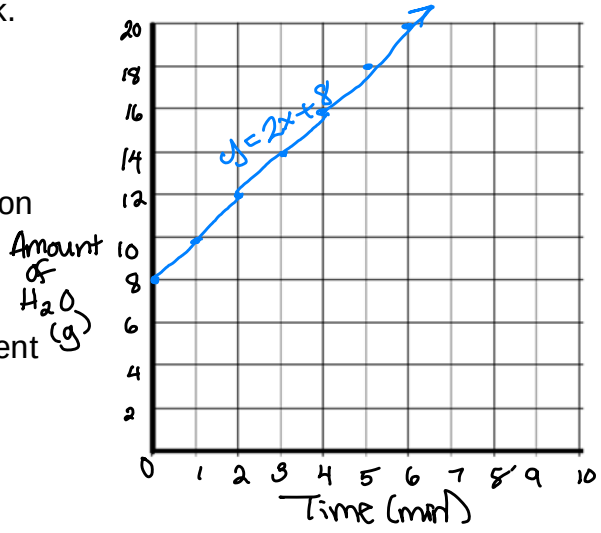
$y = mx + b$

$y = 2x + 8$

b. Construct a table of x and y values for the function

x	0	1	2	3	4
y	8	10	12	14	16

c. Use the table of values to plot a graph to represent the function.



Notes 8.2 – Linear vs. Nonlinear Functions

How do you find out if a function is linear or nonlinear?

A function is linear if the rate of change (m) is constant

Identify if the following functions are linear or nonlinear.

1.)

x	1	2	4	7	8
y	18	12	0	−18	−24

Δx = 1, Δx = 2, Δx = 3, Δx = 1

Δy = −6, Δy = −12, Δy = −18, Δy = −6

$\frac{-12}{2} = -6$, $\frac{-18}{3} = -6$, $\frac{-6}{1} = -6$

$\frac{\Delta y}{\Delta x} = \frac{-6}{1} = -6$

This is linear b/c m is constant.

2.)

x	−5	−3	−1	1	3
y	28	26	22	14	4

Δx = −2, Δx = −2

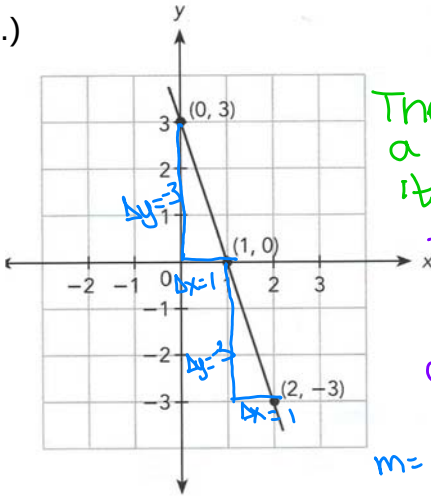
Δy = −2, Δy = −4

$m = \frac{-2}{-2} = 1$, $m = \frac{-4}{-2} = 2$

This is not linear because m is not constant.

For 3 – 4, determine if the graph is a function. Then determine if it is linear or non-linear. If it is linear, find the rate of change.

3.)

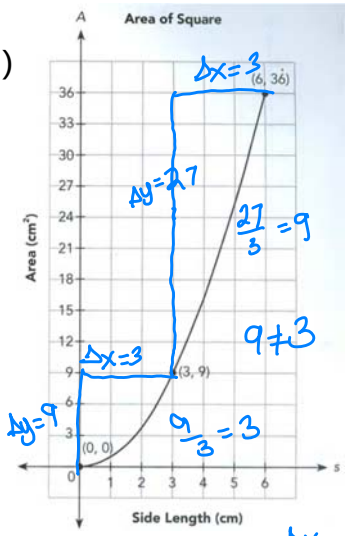


The graph is a function b/c it passes the VLT.

The function is linear because it's a straight line and m is constant. (−3).

$m = \frac{\Delta y}{\Delta x} = -3$

4.)



The graph is a function b/c it passes the VLT.

The function is not linear because it's not a straight line and m is not constant.

5.) The table below shows the total distance, y miles, indicated on the odometer of Ariel's car and the amount of gasoline used, x gallons, on a particular day.

Amount of Gasoline (x gallons)	0	1	2	3	4	5
Total Distance (y miles)	1,000	1,030	1,060	1,090	1,120	1,150

Δx = 1, Δx = 1, Δx = 1, Δx = 1

Δy = 30, Δy = 30, Δy = 30, Δy = 30

starting amount b = 1000

$m = \frac{30}{1} = 30$ mpg

a. Write an algebraic equation for the function.

$y = mx + b$

$y = 30x + 1000$

b. Describe how the slope and the y-intercept of the graph are related to the function.

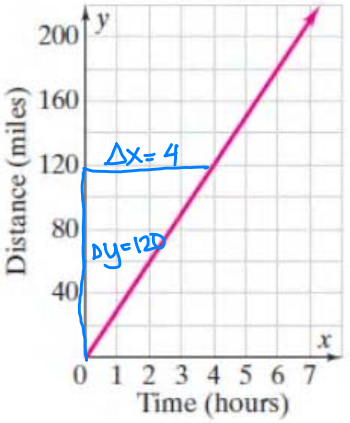
The car started with 1000 g as the total distance (y-intercept) and the total distance is increasing at a rate of 30 miles for every gallon of gas.

Notes 8.3 – Comparing Functions Day 1

Example 1

Three trains (A, B, and C) leave a train station at the same time. The graph shows the relationship between time and distance for Train A.

Train A



Train B

$y = 45x$

Train C

Time (hours)	Distance (miles)
3	105
6	210
9	315
12	420

Δx = 3, Δy = 105
Δx = 3, Δy = 105
Δx = 3, Δy = 105

1.) What is the slope of the graph?

$m = \frac{\Delta y}{\Delta x} = \frac{120}{4} = 30$ mph

2.) What does this slope represent?

The train is traveling at a rate of 30 miles per hour.

3.) The relationship between time and distance for Train B is given by the equation above, where x represents hours and y represents miles. Find the slope m.

$y = mx + b$
$y = 45x$
$m = 45$

The train is traveling at a rate of 45 mph.

4.) Which train is moving faster, Train A or Train B? How do you know?

Train B is traveling faster because it has a larger rate (slope).

5.) The time-distance relationship for Train C is shown in the table above. What is the ratio of distance to time?

$m = \frac{\Delta y}{\Delta x} = \frac{105}{3} = 35$ mph

6.) Compare the speed of Train C to the speeds of Train A and Train B.

Train C is faster than train A but slower than Train B.

Example 2

Water is pumped into two aquariums, P and Q. The tables show two functions relating the total amount of water, y liters, and the time taken, t minutes, to pump the water into each aquarium.

Aquarium P

Time Taken (t minutes)	5	10	20	30
Total Amount of Water (y liters)	70	120	220	320

$y = 10t + 20$

Aquarium Q

Time Taken (t minutes)	5	10	20	30
Total Amount of Water (y liters)	95	170	320	470

$y = 15t + 20$

1.) Write an algebraic equation to represent each function.

Aquarium P: $y = 10t + 20$

Aquarium Q: $y = 15t + 20$

2.) Which of the two aquariums is filled with water more quickly? Explain.

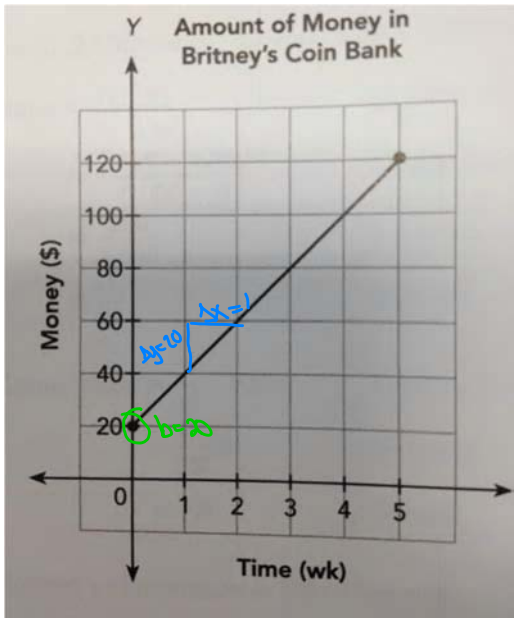
Aquarium Q has a higher rate. It's being filled up at a rate of 15 gl/min, while Aquarium P is filling up at a rate of 10 gl/min.

3.) Which of the two aquariums started with more water? Explain.

They both started with 20 gallons of water (they both have a y-intercept of 20 gallons)

Notes 8.4 – Comparing Functions Day 2

1.) Britney and Dina each have a bank account. Britney starts with a certain amount of money and adds money at regular intervals. Dina starts with a different amount of money and takes money out over time. The amount of money, y dollars, in Dina's coin bank after x weeks is given by the equation $y = -30x + 120$. The graph shows the amount of money in Britney's coin bank after x weeks.



a.) Find the y-intercept of Britney's graph and explain what information it gives about the situation.

The y-intercept is \$20. Britney starts with \$20 in her coin bank.

b.) Find the slope of Britney's graph and explain what information it gives about the situation.

$m = \frac{\Delta y}{\Delta x} = \frac{20}{1} = \$20/wk.$

Britney is adding \$20 into her bank each week

c.) What is Britney's equation?

$y = 20x + 20$

d.) Is Isa (Britney) adding money at a faster rate or is Taylor (Dina) taking out money at a faster rate? Explain.

Dina is taking money at a rate of \$30/wk (the slope in her equation), while Britney is adding money at a rate of \$20/wk. So, Dina's rate is faster.

e.) After how many weeks will they have the same amount of money in their bank accounts?

Dina: $y = -30x + 120$
Britney: $y = 20x + 20$
} want y to be equal (substitution)

$-30x + 120 = 20x + 20$
$+30x \quad +30x$
$120 = 50x + 20$
$-20 \quad -20$
$100 = 50x$
$2 = x$

2 weeks

2.) Which function has a greater rate of change? → larger slope.

Function A

x	1	3	4	6
y	5	13	17	25

$\Delta x = 2$, $\Delta x = 1$, $\Delta x = 2$

$\Delta y = 8$, $\Delta y = 4$, $\Delta y = 8$

$m = \frac{8}{2} = 4$

$m = \frac{4}{1} = 4$

Function B

$2y - 2 = 4x$

$+2 \quad +2$

$\frac{2y}{2} = \frac{4x}{2} + \frac{2}{2}$

$y = 2x + 1$

$m = 2$

Function A has a greater rate of change.

3.) The functions below represent stock prices each week, where w is the number of weeks and c is the cost. Order the stock prices from least to greatest based on their rate of change.

Alpha

$b = \$54$

The starting price of $54 decreases weekly by $2.50

$m = -2.50$

$y = -2.5x + 54$

$C = -2.5x + 54$

$m = -2.5$

Beta

$9w + 2c = 54$

$-9w \quad -9w$

$\frac{2c}{2} = \frac{54}{2} - \frac{9w}{2}$

$C = 27 - 4.5w$

$m = -4.5$

Delta

w	0	2	4	6
c	\$24	\$17	\$10	\$3

$\Delta x = 2$, $\Delta x = 2$, $\Delta x = 2$

$\Delta y = -7$, $\Delta y = -7$, $\Delta y = -7$

$m = \frac{-7}{2}$

$m = -3.5$

Rate of change from least to greatest:

Alpha, Delta, Beta

Look at absolute value of rate to determine answers.