## Unit 8 Problen Set Packet - Functions

Name: $\qquad$ Class: $\qquad$
*PS 8.1 - Introduction to Functions and Function Notation*
1.) Function rules do not always have to be numerical in nature, they simply have to return a single output for a given input. The table below gives a rule that takes as an input a neighborhood child and gives as an output the month he or she was born in.
a.) Why can we consider this rule a function?
b.) What is the output when the input is Cassidy?

| Child | Birth Month |
| :--- | :--- |
| Sabrina | January |
| Grace | April |
| Leah | May |
| Cassidy | February |
| Andrew | May |

c.) Find all inputs that give an output of May. Why does this not violate the definition of a function even though there are two answers?
2.) Megan heads out to school by foot on a fine spring day. Her distance from school, in blocks, is given as a function of the time, in minutes, she has been walking. This function is represented by the graph given below.
a.) How far does Megan start off from school?
b.) What is her distance from school after she has been walking for 4 minutes?
c.) After walking for six minutes, Megan stops to look for her subway pass. How long does she stop for?

e.) How long did it take for her to get to school once she got on the train?
d.) Charlene then walks to a subway station before heading to school on the subway (a local). How many blocks did she walk to the subway?
3.) Decide whether each of the following relations is a function. Explain your answer. Input Outputs Function? (include explanation)
a.) States
b.) States
c.) Families

Pets
d.) Families

Last Names
4.) In each of the following examples, use an input-output chart to decide if the following relation is a function.
a.) Consider the following relation: multiply
c.) Consider the following table; the input by five and then subtract seven to get the output.

| Input <br> $x$ | Calculation | Output |
| :---: | :---: | :---: |
| $y$ |  |  |$|$| -3 |  |
| :---: | :---: |
| 0 |  |
| 6 |  |

Function? Yes/No Explain.
b.) Consider the following graph.


| Input <br> $x$ | Calculation | Output(s) <br> $y$ |
| :---: | :--- | :---: |
| -2 | None |  |
| 1 | None |  |
| 2 | None |  |

Function? Yes/No Explain.

| Input <br> $\boldsymbol{x}$ | Calculation | Output <br> $y$ |
| :---: | :---: | :---: |
| -2 | None | 4 |
| 3 | None | 3 |
| 3 | None | 2 |

Function? Yes/No Explain.
d.) Consider the following graph.


| Input <br> $x$ | Calculation | Output(s) <br> $y$ |
| :---: | :--- | :---: |
| -3 | None |  |
| 1 | None |  |
| 3 | None |  |

Function? Yes/No Explain.
5.) Nathan has a new job at the local pizza store as a delivery boy. The following graph shows one of his deliveries he made. Analyze the graph and answer the following questions.
a.) How long was the entire trip?
b.) If he arrived at the house after 4 minutes, how far away was the house from the pizza place?
c.) Why might Nathan have stopped 3 times for 1 minute?

d.) Was Nathan's trip longer going to the house or

Time Spent Driving coming back?
6.) Given the following scenario, graph a function that would map Amanda's distance away from her house according to the time elapsed.

Amanda has a few items she needs to pick up from a grocery store 8 blocks away. Amanda travels as a constant rate of 2 blocks per minute when not stopped at a light. On her way to the grocery store she doesn't hit any red lights and the trip takes her 4 minutes. She spends 8 minutes in the grocery store and then starts to head home. When she's halfway home she hits a red light that lasts 3 minutes. After the light ends, she then drives the second half of the way home.

7.) If $f(x)=2 x^{2}+3 x-4$, find:
a.) $f(0)$
b.) $f(2)$
c.) $f(-2)$
d.) $f(x-5)$
8.) If the function $f(x)=2 x-3$ and $g(x)=\frac{3}{2} x+1$, then which of the following is a true statement?
(A) $f(0)>g(0)$
(C) $f(8)=g(8)$
(B) $f(2)=g(2)$
(D) $g(4)<f(4)$
9.) Based on the graph of the function $y=g(x)$ shown below, answer the following questions.
a.) Evaluate each of the following. Illustrate with a point on the graph.

$$
g(-2)=
$$

$g(0)=$
$g(3)=$
$g(7)=$
b.) What values of $x$ solve the equation $g(x)=0$ ? These are called the zeroes of the function.
c.) How many values of $x$ solve the equation $g(x)=2$ ? How can you illustrate your answer on the graph.


Remember, we are not looking for the exact $x$-values, only how many solutions.
10.) Physics students drop a ball from the top of a 100 foot high building and model its height above the ground as a function of time with the equation $h(t)=100-16 t^{2}$. The height, $h$, is measured in feet and time, $t$, is measured in seconds. Be careful with all calculations in this problem and remember to follow order of operations.
a.) Find the value of $h(0)$. Include proper units. What does this output represent?
b.) Calculate $h(2)$. Does our equation predict that the ball has hit the ground at 2 seconds? Explain.
11.) If you knew that $f(-4)=8$, then what $(x, y)$ coordinate point must lie on the graph of $y=f(x)$ ? Explain your thinking.
*PS 8.2 - Graphs of Functions and Their Features
12.) Using the graph of the function $f(x)$ shown below, answer the following questions.
a.) Find the value of each of the following:

$$
\begin{array}{ll}
f(-7)= & f(0)= \\
f(4)= & f(9)=
\end{array}
$$

b.) For how many values of $x$ does $f(x)=5$ ? Illustrate on the graph.

c.) What is the $y$-intercept of this relation?
d.) State the maximum and minimum values the graph obtains.
e.) Explain why the graph above represents a function.
13.) Consider the function $f(x)=3(2-x)-2$. Fill out the function table below for the inputs given and graph the function on the axes provided.

| $x$ | $3(2-x)-2$ | $(x, y)$ |
| :--- | :--- | :--- |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |


14.) The following graph represents the cost of a phone plan after a certain number of text messages used in a month. Analyze the graph to answer the following questions.
a.) How much would you have to pay if you used:

500 text messages $\qquad$

1800 text messages $\qquad$
b.) Interpret $f(1400)=60$

c.) What might have caused the graph to begin increasing at 800 text messages?
15.) Evaluate each function.
a.) $a(x)=x^{2}-3 x$; Find $a(-8)$
b.) $p(r)=-4^{3 r}$; Find $p(-1)$
16.) Consider the following relationship given by the formula $f(x)=\left\{\begin{array}{ll}3-2 x & x \leq 1 \\ 2 x-1 & x>1\end{array}\right.$.
a.) Evaluate each of the following:
$f(5)=$
$f(-2)=$
b.) Carefully evaluate $f(1)$.
c.) Fill out the table below for the inputs given. Keep in mind which formula you are using.

| $x$ | Rule/Calculation | $(x, y)$ |
| :---: | :---: | :---: |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

17.) The following graph shows the height, $h$, above the ground of a toy rocket $t$ seconds after it was fired. Use the graph of $h(t)$ to answer the following questions.
a.) What was the maximum height the rocket reached?
b.) How many seconds was the rocket in flight?
c.) Interpret $h(2)=90$.
d.) Give the interval for $t$ over which the height of the rocket is decreasing.
d.) Graph $y=f(x)$ on the axes below.

e.) What is the minimum value of the function? Circle the point that indicates this value on the graph.

$t$, time elapsed
(in seconds)
18.) On the following set of axes, create the graph of a function $f(x)$ with the following characteristics.

Passes through the points,
$(-8,0),(5,-2)$, and $(8,3)$
Has an absolute maximum at $f(-4)=5$

Has an absolute minimum at $f(2)=-6$
Decreasing on the interval $-4 \leq x \leq 2$


## *PS 8.3 - Average Rate of Change*

19.) Consider the function given by $f(x)=x^{2}+3$. Find its average rate of change from $x=-1$ and $x=3$. Carefully show the work that leads to your final answer.
20.) Function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \leq x \leq 6$. Show the calculations that lead to your answer.
(A) $-\frac{3}{2}$
(C) $-\frac{7}{6}$
(B) $\frac{6}{4}$
(D) -1

| $x$ | $h(x)$ |
| :---: | :---: |
| 0 | 10 |
| 2 | 9 |
| 4 | 6 |
| 6 | 3 |

21.) Jake is selling glasses of lemonade. The function $g(t)=\frac{t^{2}+4}{2}$ models the number of glasses he has sold, $g$, after $t$-hours. What is the average rate at which he is selling lemonade between $t=2$ and $t=8$ hours. Include proper units in your answer.
22.) Consider the function given by $f(x)=9-x^{2}$. Find its average rate of change between the following points. Carefully show the work that leads to your final answer.
a.) $x=0$ to $x=3$
b.) $x=-1$ to $x=5$
c.) $x=-2$ to $x=2$
23.) The function $g(x)$ is given in the table below. Find its average rate of change between the following points. Show the calculations that lead to your answer.
a.) $x=-3$ to $x=1$
b.) $x=0$ to $x=4$

| $x$ | $g(x)$ |
| :---: | :---: |
| -3 | 7 |
| 0 | -2 |
| 1 | 3 |
|  | -8 |

24.) The function $h(x)$ is given in the graph below. Find its average rate of ${ }^{4}$ change between the following points. Show the calculations that lead to your answer.
a.) $x=-6$ to $x=4$
b.) $x=-2$ to $x=2$

25.) The following table shows the number of points the Arlington girls team scored in their last basketball game where $t$ is the time passed in minutes and $f(t)$ is the total number of points scored after $t$ minutes.
a.) What was the average rate they were shooting the first half of the game? Be sure to include proper units in your answer.

| $t$ | $f(t)$ |
| :---: | :---: |
| 0 | 0 |
| 8 | 30 |
| 16 | 48 |
| 24 | 55 |
| 32 | 64 |

b.) What was their average rate over the whole game?
c.) Given your answers above, which half of the game do you feel they had a better rate of scoring? Justify your answer.
26.) Consider the function given by $f(x)=6 x+5$.
a.) Find its average rate of change from $x=1$ to $x=5$.
b.) Find its average rate of change from $x=-2$ to $x=6$.
c.) Find its average rate of change from $x=3.8$ to $x=9.4$.
d.) The average rate of change for this function is always 6 (as you have found in the first three parts of the problem). What type of function has a constant rate of change? What do we call the average rate of change in this case?
*PS 8.4 - Domain and Range*
27.) In each of the following, state the domain and range; then decide if it's a function or not. Be sure to explain using words such as input, output, domain and range!
a.)


Domain: $\qquad$
Range: $\qquad$
Function (yes/no):
b.)

Domain Range


Domain: $\qquad$
Range: $\qquad$
Function (yes/no):
28.) Consider the piecewise linear function given by the formula $f(x)=\left\{\begin{array}{cc}2-3 x & -1 \leq x \leq 1 \\ x-2 & 1<x \leq 3\end{array}\right.$.
Determine the function's domain and range. Draw a graph of the function to fully justify your answer. Use tables on your calculator to help graph.

29.) The following graph represents the height above the ground versus time at a resort as Jensen rides his favorite ski slope.
(a) State the domain and, in your own words, what the domain represents.
(b) State range and, in your own words, what the range represents.

(c) What might Jensen have been doing for the interval $0 \leq t \leq 2$ ? What was his average rate of change? Use proper units in your answer.
(d) What might Jensen have been doing for the interval $2 \leq t \leq 6$ ? What was his average rate of change? Use proper units in your answer and compare to what you found in (c).
30.) The graph below represents the height of a ball over the interval $0 \leq t \leq 8$. After how many seconds was the ball 12 feet off of the ground? Explain your answer.


