Unit 9 Problem Set Packet - Exponential Functions

Name: ____ Class: ________________________

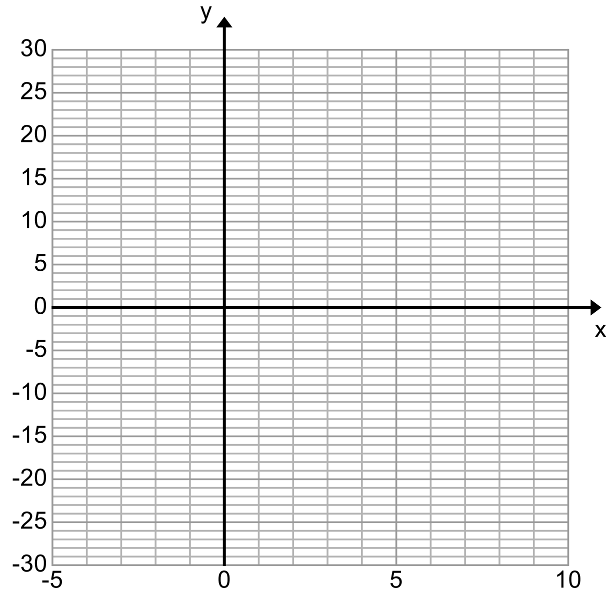
Integer Sequences and Recursive Formulas

1. Consider a sequence that follows a "minus 5" pattern: $30, 25, 20, 15, \ldots.$

   Write a formula for the nth term of the sequence. Be sure to specify what value of n your formula starts with.

   Using the formula, find the 20th term of the sequence.

   Graph the terms of the sequence as ordered pairs $(n, f(n))$ on a coordinate plane.



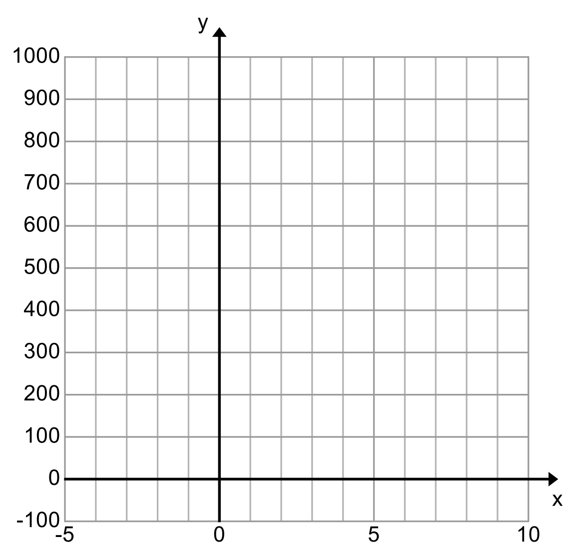
   Does this relationship appear to be linear, quadratic, or exponential?

2. Consider a sequence that follows a "times 5" pattern: $1, 5, 25, 125, \ldots.$

   Write a formula for the $\boldsymbol{n}$th term of the sequence. Be sure to specify what value of n your formula starts with.

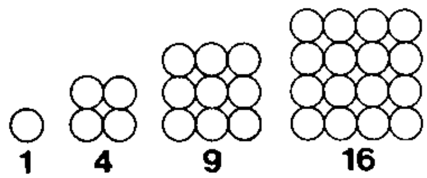
   Using the formula, find the 10th term of the sequence.

   Graph the terms of the sequence as ordered pairs $(\boldsymbol{n}, \boldsymbol{f}(\boldsymbol{n}))$ on a coordinate plane.



   Does this relationship appear to be linear, quadratic, or exponential?

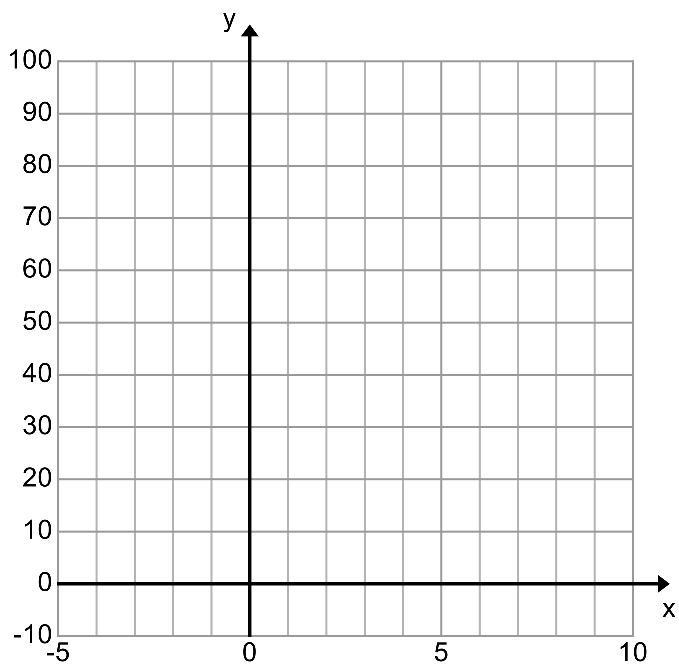
3. Consider the sequence formed by the square numbers:



Write a formula for the nth term of the sequence. Be sure to specify what value of n your formula starts with.

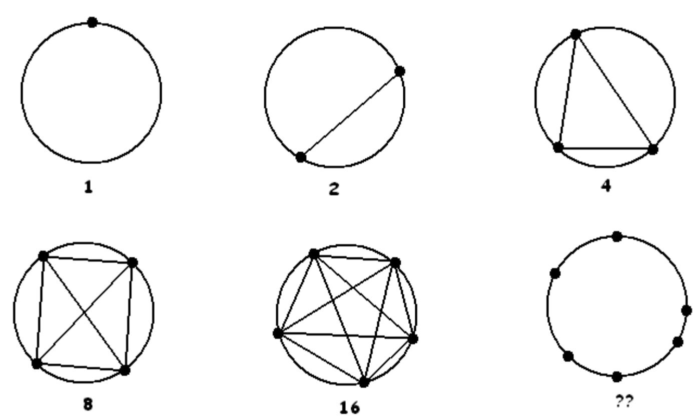
Using the formula, find the 50th term of the sequence.

Graph the terms of the sequence as ordered pairs $(n, f(n))$ on a coordinate plane.



Does this relationship appear to be linear, quadratic, or exponential?

4. Here is the classic puzzle that shows that patterns need not hold true. What are the numbers counting?



a. Based on the sequence of numbers, predict the next number.

b. Write a formula based on the perceived pattern.

c. Find the next number in the sequence by actually counting.

d. Based on your answer from c, is your model from b effective for this puzzle?

In problems 5 - 6, consider the sequence and then:

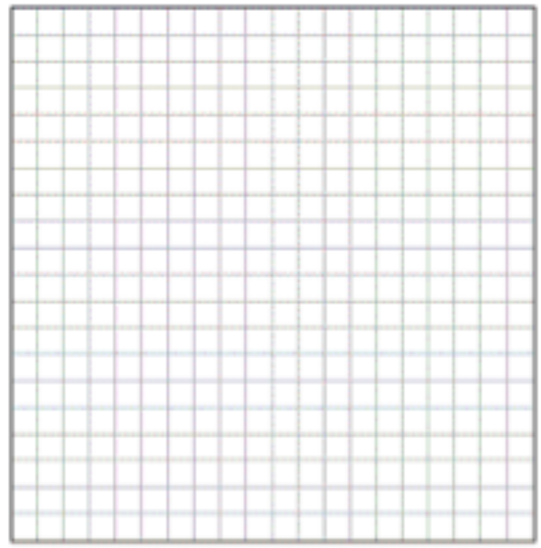
a. Write a formula for the nth term of the sequence. Be sure to specify what value of n your formula starts with.
b. Using the formula, find the 15th term of the sequence.
c. Graph the terms of the sequence as ordered pairs $(n, f(n))$ on a coordinate plane.
d. Determine if the relationship is linear, quadratic, or exponential.

5. The sequence follows a "plus 2" pattern: $3, 5, 7, 9, \ldots.$

a.

b.

c.



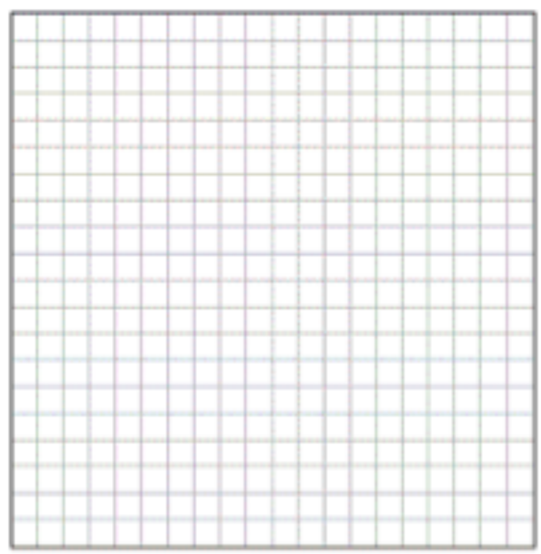
d.

6. The sequence follows a "times 4" pattern: $1, 4, 16, 64, \ldots.$

a.

b.

c.



d.

7. One of the most famous sequences is the Fibonacci sequence:

$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots.$

$$f(n+1) = f(n) + f(n-1), \text{ where } f(1) = 1, f(2) = 1, \text{ and } n \geq 2.$$

How is each term of the sequence generated?

8. Consider the sequence following a "minus 8" pattern: 9, 1, -7, -15, …

   a. Write an explicit formula for the sequence.

   b. Write a recursive formula for the sequence.

   c. Find the 38th term of the sequence.

For 9 – 10, an explicit formula is given.

a. Write the first 5 terms of each sequence.

b. Then, write a recursive formula for the sequence.

9. $a_n = 2n + 10 \text{ for } n \geq 1$

   a.

   b.

10. $a_n = \left(\frac{1}{2}\right)^{n-1} \text{ for } n \geq 1$

   a.

   b.

11. For each sequence, write *either* an explicit or recursive formula.

   a. $1, -1, 1, -1, 1, -1, \ldots$

   b. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

12. Tommy opens a bank account. The deal he makes with his mother is that if he doubles the amount that was in the account at the beginning of each month by the end of the month, she will add an additional $5 to the account at the end of the month.
    a. Let $A(n)$ represent the amount in the account at the beginning of the nth month. Assume that he does, in fact, double the amount every month. Write a recursive formula for the amount of money in his account at the beginning of the nth month.

    b. What is the least amount he could start with in order to have $\$300$ by the beginning of the 3rd month?

13. Consider the sequence given by the formula $a(n+1)=5\bullet a(n)$ and $a(1)=2$ for $n\geq 1$.
    a. Explain what the formula means.

    b. List the first 5 terms of the sequence.

For problems 14 – 17, list the first five terms of each sequence.

14. $a_{n+1} = a_n + 6$, where $a_1 = 11$ for $n \geq 1$

15. $a_n = a_{n-1} \div 2$, where $a_1 = 50$ for $n \geq 2$

16. $f(n+1) = -2f(n) + 8$ and $f(1) = 1$ for $n \geq 1$

17. $f(n) = f(n-1) + n$ and $f(1) = 4$ for $n \geq 2$

For problems 18 – 23, write a recursive formula for each sequence given or described below.

18. It follows a "plus one" pattern: $8, 9, 10, 11, 12, \ldots$.

19. It follows a "times 10" pattern: $4, 40, 400, 4000, \ldots$.

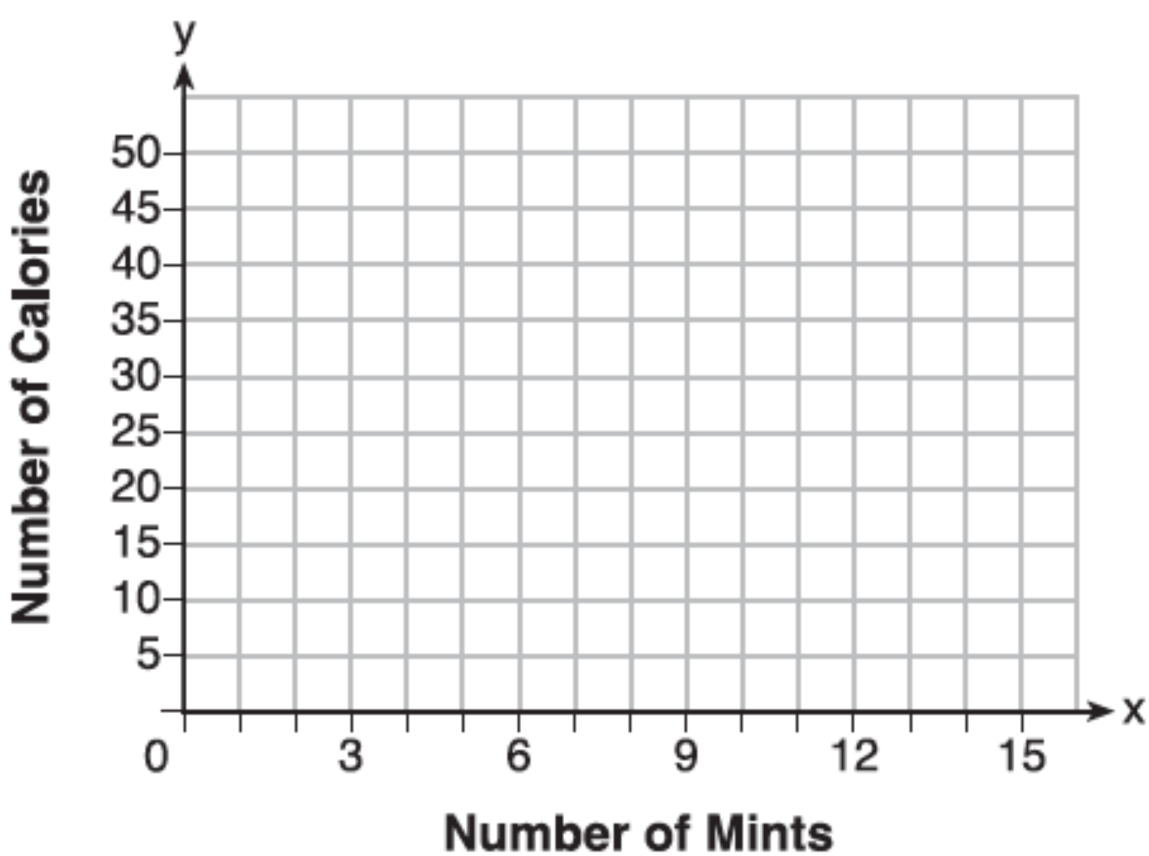
20. It has an explicit formula of $f(n) = -3n + 2$ for $n \geq 1$.

21. It has an explicit formula of $f(n) = -1(12)^{n-1}$ for $n \geq 1$.

22. Doug accepts a job where his starting salary will be $30,000 per year, and each year he will receive a raise of $3,000.

23. A bacteria culture has an initial population of 10 bacteria, and each hour the population triples in size.

24. Max purchased a box of green tea mints. The nutrition label on the box stated that a serving of three mints contains a total of 10 Calories. On the axes below, graph the function, C, where C9x) represents the number of Calories in x mints.



Write an equation that represents C(x).

A box full of mints contains 180 Calories. Use the equation to determine the total number of mints in the box.

Arithmetic and Geometric Sequences

For problems 33 – 36, list the first five terms of each sequence, and identify them as arithmetic or geometric.

25. $\boldsymbol{A(n+1) = A(n) + 4}$ for $\boldsymbol{n \geq 1}$ and $\boldsymbol{A(1) = -2}$

26. $\boldsymbol{A(n+1) = \frac{1}{4} \cdot A(n)}$ for $\boldsymbol{n \geq 1}$ and $\boldsymbol{A(1) = 8}$

27. $\boldsymbol{A(n+1) = A(n) - 19}$ for $\boldsymbol{n \geq 1}$ and $\boldsymbol{A(1) = -6}$

28. $\boldsymbol{A(n+1) = \frac{2}{3} A(n)}$ for $\boldsymbol{n \geq 1}$ and $\boldsymbol{A(1) = 6}$

For problems 5–8, identify the sequence as arithmetic or geometric, and write a recursive formula for the sequence. Be sure to identify your starting value.

29. $14,\ 21,\ 28,\ 35, \ldots$

30. $4,\ 40,\ 400,\ 4000, \ldots$

31. $49,\ 7,\ \frac{1}{7}, \frac{1}{49}, \ldots$

32. $-101,\ -91,\ -81,\ -71, \ldots$

33. The local football team won the championship several years ago, and since then, ticket prices have been increasing $20 per year. The year they won the championship, tickets were $50. Write a recursive formula for a sequence that will model ticket

prices. Is the sequence arithmetic or geometric?

34. A radioactive substance decreases in the amount of grams by one third each year. If the starting amount of the substance in a rock is $\mathbf{1,452}$ g, write a recursive formula for a sequence that models the amount of the substance left after the end of each year. Is the sequence arithmetic or geometric?

35. Find an explicit form $f(n)$ for each of the following arithmetic sequences (assume a is some real number and x is some real number):

a. $\mathbf{-34,\ -22,\ -10,\ 2, \ldots}$

b. $\mathbf{\frac{1}{5},\ \frac{1}{10},\ 0,\ -\frac{1}{10}, \ldots}$

c. $\boldsymbol{x+4,\ x+8,\ x+12,\ x+16, \ldots}$

d. $\boldsymbol{a,\ 2a+1,\ 3a+2,\ 4a+3, \ldots}$

36. Consider the arithmetic sequence $13,\ 24,\ 35, \ldots.$

a. Find an explicit form for the sequence in terms of $\boldsymbol{n}$.

b. Find the 40th term.

c. If the $\boldsymbol{n}$th term is $\mathbf{299}$, find the value of $\boldsymbol{n}$.

37. If $-2,\ a,\ b,\ c,\ 14$ forms an arithmetic sequence, find the values of a, b, and c.

38. Find the common ratio and an explicit form in each of the following geometric sequences:

a. $\mathbf{4,\ 12,\ 36,\ 108, \dots}$

b. $\mathbf{162,\ 108,\ 72,\ 48, \dots}$

c. $\mathbf{\frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \dots}$

d. $\boldsymbol{xz,\ x^2z^3,\ x^3z^5, x^4z^7, \dots}$

39. The first term in a geometric sequence is 54, and the 5th term is $\frac{2}{3}$. Find an explicit form for the geometric sequence.

40. If 2, a, b, -54 forms a geometric sequence, find the values of a and b.

Challenge Questions:

41. $3 + x, 9 + 3x, 13 + 4x, \ldots$ is an arithmetic sequence for some real number x.
    a. Find the value of $\boldsymbol{x}$.

    b. Find the 10th term of the sequence.

42. Find an explicit form $\mathrm{f(n)}$ of the arithmetic sequence where the 2nd term is 25 and the sum of the 3rd term and 4th term is 86.

43. Find the explicit form $f(n)$ of a geometric sequence if $f(3) - f(1) = 48$ and $\frac{f(3)}{f(1)} = 9$.

Interest and Exponential Growth

44. $250 is invested at a bank that pays 7% simple interest. Calculate the amount of money in the account after:
    a. 1 year
    b. 7 years

45. $325 is borrowed from a bank that charges 4% interest compounded annually. Find out how much is owed after:
    a. 3 years
    b. 20 years

46. Kevin has $10,000 to invest. He can go to Yankee Bank that pays 5% simple interest or Met Bank that pays 4% interest compounded annually. After how many years will Met Bank be the better choice? (Hint: Make a chart.)

47. A three-bedroom house in Burbville was purchased for $190,000. Housing prices are expected to increase 1.8% annually in that town.

a. Write an explicit formula that models the price of the house in t years.

b. Find the price of the house in 5 years.

48. In 2013, a research company found that smartphone shipments (units sold) were up 32.7% worldwide from 2012, with an expectation for the trend to continue. 959 million units were sold in 2013.

a. Write an explicit formula that models the number of smartphone shipments in t years.

b. How many smartphones can be expected to be sold in 2018 at the same rate?

c. Can this trend continue? Why or why not?

49. A bucket is put under a leaking ceiling. The amount of water in the bucket doubles every minute. After 8 minutes, the bucket is full. After how many minutes if the container half full?

50. On June 1, a fast-growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered, and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

 a. When will the lake be covered half way?

 b. On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?

 c. On June 29, a cleanup crew arrives at the lake and removes all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

51. The table below represents the population of the state of New York for the years 1800 – 2000. Use the information to answer the questions.

Year	Population
1800	300,000
1900	7,300,000
2000	19,000,000

 a. Using the year 1800 as the base year, an explicit formula for the sequence that models the population of New York is $\mathrm{P(t)} = 300000(1.021)^{\mathrm{t}}$, where t is the number of years after 1800.
 Using this formula, calculate the projected population of New York in 2010.

 b. Using the year 1900 as the base year, an explicit formula for the sequence that models the population of New York is $\mathrm{P(t)} = 7300000(1.0096)^{\mathrm{t}}$, where t is the number of years after 1900.
 Using this equation, calculate the projected population of New York in 2010.

 c. Using the internet (or some other source), find the population of the state of New York according to the 2010 census. Which formula yielded a more accurate prediction of the 2010 population?

Exponential Decay

1. A huge ping-pong tournament is held in Beijing, with 65,536 participants at the start of the tournament. Each round of the tournament eliminates half the participants.
    d. If p(r) represents the number of participants remaining after r rounds of play, write a formula to model the number of participants remaining.

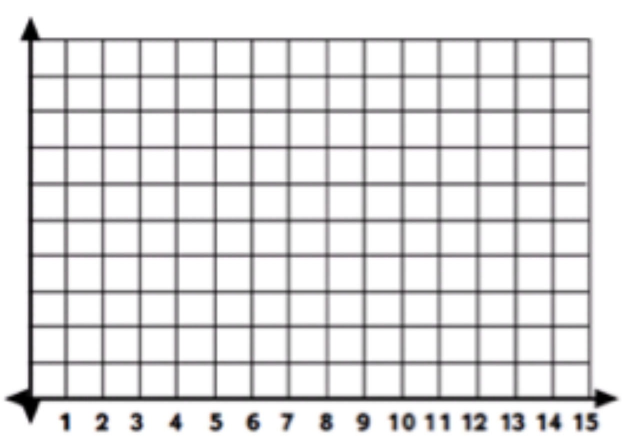
    e. Use your model to determine how many participants remain after 10 rounds of play.

    f. How many rounds of play will it take to determine the champion ping-pong player?

2. A construction company purchased some equipment costing $\$300,000$. The value of the equipment depreciates (decreases) at a rate of 14% per year.
    g. Write a formula that models the value of the equipment.

    h. What is the value of the equipment after 9 years?

    i. Graph the points $(t, v(t))$ for integer values of $0 \leq t \leq 15$.



j. Estimate when the equipment will have a value of $\$50,000$.

3. The number of newly reported cases of HIV (in thousands) in the United States from 2000 to 2010 can be modeled by the following formula:

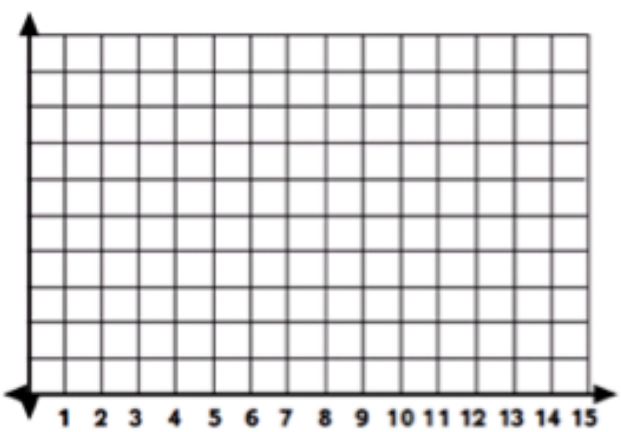
$f(t) = 41(0.9842)^t$, where t is the number of years after 2000.

k. Identify the growth factor.

l. At what rate is the number of new reported cases decreasing?

m. Calculate the estimated number of new HIV cases reported in 2004.

n. Graph the points $(t, f(t))$ for integer values of $0 \le t \le 10$.



o. During what year did the number of newly reported HIV cases drop below 36,000?

4. Doug drank a soda with 130 mg of caffeine. Each hour, the caffeine in the body diminishes by about 12%.

p. Write formula to model the amount of caffeine remaining in Doug's system.

q. How much caffeine remains in Doug's system after 2 hours?

r. How long will it take for the level of caffeine in Doug's system to drop below 50 mg?

5. 64 teams participate in a softball tournament in which half the teams are eliminated after each round of play.
   s. Write a formula to model the number of teams remaining after any given round of play.

   t. How many teams remain in play after 3 rounds?

   u. How many rounds of play will it take to determine which team wins the tournament?

6. Sam bought a used car for $\$8{,}000$. He boasted that he got a great deal since the value of the car two years ago (when it was new) was $\$15{,}000$. His friend, Derek, was skeptical, stating that the value of a car typically depreciates about 25% per year, so Sam got a bad deal.
   v. Use Derek's logic to write a formula for the value of Sam's car. Use t for the total age of the car in years.

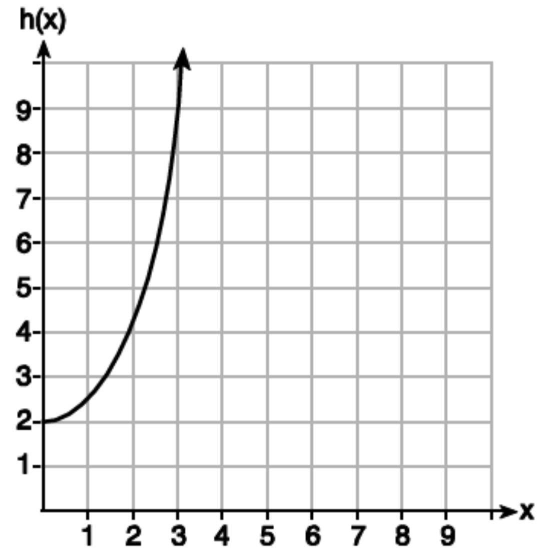
   w. Who is right, Sam or Derek? Explain.

7. For which function defined by a polynomial are the zeros of the polynomial -4 and -6?
   (A) $y = x^2 - 10x - 24$
   (B) $y = x^2 + 10x + 24$
   (C) $y = x^2 + 10x - 24$
   (D) $y = x^2 - 10x + 24$

8. Given the functions g(x), f(x), and h(x) shown below:

$g(x) = x^2 - 2x$

x	f(x)
0	1
1	2
2	5
3	7



The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \le x \le 3$ is

(A) f(x), g(x), h(x) (B) h(x), g(x), f(x) (C) g(x), f(x), h(x) (D) h(x), f(x), g(x)

9. Donna wants to make trail mix made up of almonds, walnuts, and raisins. She wants to mix one part almonds, two parts walnuts, and three part raisins. Almonds cost $12 per pounds, walnuts cost $9 per pound and raisins cost $5 per pound. Donna has $15 to spend on the trail mix. Determine how many pounds of trail mix she can make.

Answers to Unit 9 Problem Set Packet - Exponential Functions

1. $a_n = 35 - 5n$ $a_{20} = -65$ linear	2. $a_n = 5^{n-1}$ $a_{10} = 1953125$ exponential	3. Extra
4. Extra	5. Extra	6. Extra
7. Each term is generated by adding the two previous terms.	8. a. $a_n = 17 - 8n$ b. $a_{n+1} = a_n - 8,\ a_1 = 9$ c. $a_{38} = -287$	9. a. 12, 14, 16, 18, 20 b. $a_{n+1} = a_n + 2, a_1 = 12$
10. a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ b. $f(n+1) = \frac{1}{2} \bullet f(n), f(1) = 1$	11. a. $a_n = (-1)^{n-1}$ or $a_{n+1} = -1 \bullet a_n, a_1 = 1$ b. $a_n = \frac{n}{n+1}$	12. a.) $A(n+1) = 2 \bullet A(n) + 5$ b. $33.13
13. a. Each term is 5 times larger than the previous term. b. 2, 10, 50, 250, 1250	14. 11, 17, 23, 29, 35	15. 50, 25, 12.5, 6.25, 3.125
16. 1, 6, -4, 16, -24	17. 6, 9, 13, 18, 24	18. $a_{n+1} = a_n + 1, a_1 = 8$
19. $a_{n+1} = 10a_n, a_1 = 4$	20. $f(n+1) = f(n) - 3, f(1) = -1$	21. $a_{n+1} = 12 \bullet a_n, a_1 = -1$
22. $a_n = a_{n-1} + 3{,}000, a_0 = 30{,}000$	23. $f(n) = 3 \bullet f(n-1), f(1) = 10$	24. a. $C(x) = \frac{10}{3}x$ b. 54 mints
25. -2, 2, 6, 10, 14	26. $8, 2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}$	27. -6, -25, -44, -63, -82
28. $6, 4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}$	29. Arithmetic $A(n+1) = A(n) + 7, A(1) = 14$	30. Geometric $a_{n+1} = 10a_n, a_1 = 4$
31. $A(n+1) = \frac{1}{7} \bullet A(n), A(1) = 49$	32. $A(n+1) = A(n) + 10, A(1) = -101$	33. $a_{n+1} = a_n + 20, a_1 = 50$
34. $a_{n+1} = \frac{2}{3}a_n, a_1 = 1452$	35. a. $f(n) = 12n - 46$ b. $a_n = \frac{3}{10} - \frac{1}{10}n$ c. $a_n = x + 4n$ d. $b_n = an + n - 1$ *Note: You can't use a_n in question d because a is already taken in the problem.	36. a. $a_n = 11n + 2$ b. 442 c. 27

37. a = 2 b = 6 c = 10	38. a. $a_n = 4 \bullet 3^{n-1}$ b. $f(n) = 162\left(\frac{2}{3}\right)^{n-1}$ c. $a_n = \frac{4}{3}\left(\frac{1}{2}\right)^{n-1}$ d. $a_n = xz\left(xz^2\right)^{n-1}$	39. $a_n = 54\left(\frac{1}{3}\right)^{n-1}$
40. a = -6 b = 18	41. Extra	42. Extra
43. Extra	44. a. $267.50 b. $372.50	45. a. $365.58 b. $712.12
46. 12 years	47. a. $f(t) = 190{,}000(1.018)^t$ b. $207,726.71	48. a. $f(t) = 959(1.327)^t$ b. 3,946,146,656 units c. No, there are a finite number of people in the world.
49. 7 minutes	50. a. June 29 b. On June 26, it was only 6.25%; it's hard to believe that in four days it would reach 100% c. Not really; by 7/6, it will be back to 100%	51. a. 464,154 b. 20,880,960 c. In 2010, there were about 19 million people. The formula from part b yielded a more accurate prediction of the 2010 population.
52. a. $p(r) = 65{,}636(0.5)^r$ b. 64 participants c. 16 rounds	53. a. $f(t) = 300000(.86)^t$ b. $77,198.23 d. After 10 years	54. a. 0.9842 b. 0.0158 (or 1.58%) c. about 38,000 new cases e. after 9 years
55. a. $f(t) = 130(.88)^t$ b. 101 mg c. 8 hours	56. a. $f(t) = 64(0.5)^t$ b. 8 teams c. 6 rounds	57. a. $f(t) = 15000(.75)^t$ b. Sam got a good deal because the car is worth $8,437.50 and he got it for $8,000.
58. B	59. D	60. 2 pounds (1/3 pound of almonds, 2/3 pound of walnuts, 1 pound of raisins)