Unit 12 Notes
Polynomials and Factoring

Tentative Schedule

Day	Class Work	Assignment
Tues. 5/26 Wed. 5/27	Introduction to Polynomials Adding and Subtracting Polynomials	P.S. #12.1
Thurs. 5/28	Multiplying and Dividing Polynomials by Monomials	P.S. #12.2
Fri. 5/29 Mon. 6/1	Multiplying Polynomials by Polynomials	P.S. #12.3
Tues. 6/2	Review of Polynomials	P.S. #12.3b
Wed. 6/3 Thurs. 6/4	Factors and GCF	P.S.#12.4
Fri. 6/5	Factoring by GCF	P.S. #12.5
Mon. 6/8 Tues. 6/9	Factoring Trinomials	P.S. #12.6
Wed. 6/10	Practice!	P.S. #12.6b
Thurs. 6/11 Fri. 6/12	Review for Test #12	Review for Test #12
Mon. 6/15	Test #12	

Name: _____

Notes 12.1 - Introduction to Polynomials
Adding and Subtracting Polynomials

*each term is separated by addition/subtraction signs

Monomial	one algebraic term Ex: $5x$ or $3x^2$
Binomial	two algebraic terms Ex: $2x^2+7x$
Trinomial	three algebraic terms Ex: $5x^3+2x^2+7x$
Polynomial	many algebraic terms Ex: $2x^4+5x^2$
Degree	the highest exponent in an expression

For 1-6, determine of the following are a monomial, binomial, or a trinomial.

1. $3x^2$
monomial

2. $4x+4$
binomial

3. $7x^2+8x+1$
trinomial

4. $8x^3+3x^2+x$
trinomial

5. $17x$
monomial

6. $3x^2-2x$
binomial

For 7-12, put the polynomial in descending order and determine the degree.

7. $3x^4+5x^2+3$
$3x^4+5x^2+3$
4th degree

8. $12x^5+2x+8$
$12x^5+2x+8$
5th degree

8. $15x+3$
$15x+3$
1st degree

10. $4x^3+2x-8x^4$
$-8x^4+4x^3+2x$
4th degree

11. $-13x+8x^4$
$8x^4-13x$
4th degree

12. $5x^3-7x^5+2x^2-4+8x$
$-7x^5+5x^3+2x^2+8x-4$
5th degree

Finding sums of polynomials:

$(3x^2 - 2x + 5) + (4x^2 + 8x - 6)$

1.) Underline each like term in a different way.

2.) Every time you have a new like term, combine them.

3.) Make sure your answer is written in descending order (standard form).

$7x^2 + 6x - 1$

1.) $(8y^2 + 2y) + (3y^2 + 7y)$

$11y^2 + 9y$

2.) $(3x^3 - 4x^2 + 4x - 2) + (7x^3 - 9x^2 + 2x + 4)$

$10x^3 - 13x^2 + 6x + 2$

3.) $(6r^4 + r + 3) + (3r^3 + 2r^2 + 7)$

$6r^4 + 3r^3 + 2r^2 + r + 10$

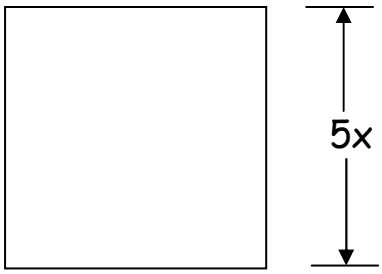
4.) $(12y^3 - 9y + 8y + 2) + (6y^2 - 12 - 3y + 2y^2)$

$12y^3 + 8y^2 - 4y - 10$

Perimeter: The distance around a figure.

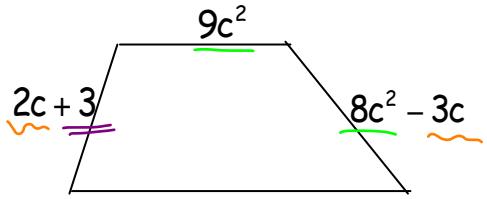
Find the perimeter of each.

5.)



$4(5x) = 20x$

6.)



$17c^2 + 16c - 1$

Finding differences of polynomials:

$$(3x^2 - 2x + 5) - (4x^2 + 8x - 6)$$

1.) Make the subtraction sign in between each expression positive and negate/change all the signs in the following expression. (You're distributing the negative)

2.) Underline each like term in a different way.

3.) Every time you have a new like term, combine them.

4.) Make sure your answer is written in descending order.

$(3x^2 - 2x + 5) + (-4x^2 - 8x + 6)$

$-x^2 - 10x + 11$

5.) $(8y^2 + 2y) - (3y^2 + 7y)$

$(8y^2 + 2y) + (-3y^2 - 7y)$

$5y^2 - 5y$

6.) $(3x^3 - 4x^2 + 4x - 2) - (7x^3 - 9x^2 + 2x + 4)$

$(3x^3 - 4x^2 + 4x - 2) + (-7x^3 + 9x^2 - 2x - 4)$

$-4x^3 + 5x^2 + 2x - 6$

7.) $(6r^4 + r + 3) - (3r^3 + 2r^2 + 7)$

$(6r^4 + r + 3) + (-3r^3 - 2r^2 - 7)$

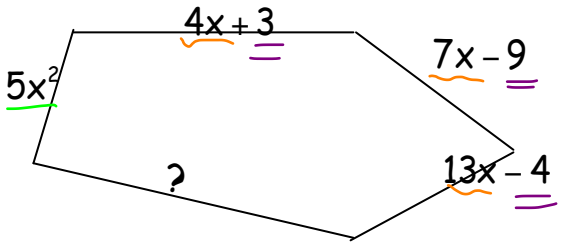
$6r^4 - 3r^3 - 2r^2 + r - 4$

8.) $(12y^3 - 9y + 8y + 2) - (6y^2 - 12 - 3y + 2y^2)$

$(12y^3 - 9y + 8y + 2) + (-6y^2 + 12 + 3y - 2y^2)$

$12y^3 - 8y^2 + 2y + 14$

5. Perimeter = $12x^2 + 25x + 8$



$(12x^2 + 25x + 8) - (5x^2 + 4x + 3 + 7x - 9 + 13x - 4)$

$(12x^2 + 25x + 8) - (5x^2 + 24x - 10)$

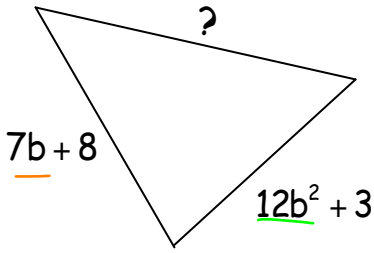
(change signs)

(distribute the negative)

$(12x^2 + 25x + 8) + (-5x^2 - 24x + 10)$

$7x^2 + x + 18$

6. Perimeter = $23b - 7$



$(23b - 7) - (12b^2 + 7b + 11)$

(Distribute the negative)

$(23b - 7) + (-12b^2 - 7b - 11)$

$-12b^2 + 16b - 18$

Notes 12.2 – Multiplying Polynomials by Monomials

Reminder:

- When you multiply monomials, you must always add exponents.

Distribute:

- When you distribute, you must multiply the number on the outside of the parentheses by EVERY number on the inside of the parentheses.

Try These:

1. $2(x + 8)$

$2x+16$

2. $c(c + 4)$

c^2+4c

3. $3d(7d + 2)$

$21d^2+6d$

4. $3a(2a^2 + 7ab)$

$6a^3+21a^2b$

5. $13x^4(2x^7 + 3)$

$26x^{11}+39x^4$

6. $9j(-3j^2 + 2j - 7)$

$-27j^3+18j^2-63j$

7. $x(x + 2) - 3x(x - 6)$

(Don't forget to distribute the negative.)

$x^2+2x-3x^2+18x$

$-2x^2+20x$

8. $x^3(x + 5) - x(x^3 - 9)$

$x^4+5x^3-x^4+9x$

$5x^3+9x$

9. If the length of a rectangle is $5L^2$ and the width of a rectangle is $7L + 2$. What is the area of the rectangle?

$5L^2(7L+2)$

$35L^3+10L^2$

You have to divide **everything** by what is in the denominator.

$\frac{10x+15}{5} = \frac{10x}{5} + \frac{15}{5}$

$= 2x+3$

Examples:

10. $\frac{2x+16}{2} = x+8$

11. $\frac{c^2+8c}{c}$

$c+8$

12. $\frac{12d^2+6d}{3d} = 4d+2$

13. $\frac{72x^3-32x^2+8x}{8x}$

$9x^2-4x+1$

14. $\frac{18r^4s-27r^3s^2}{9rs} = 2r^3-3r^2s$

15. $\frac{21c^3-12c^2+3c}{-3c}$

$-7c^2+4c-1$

16. $\frac{14a^3-105a^2b}{7a} =$

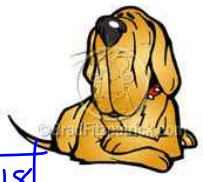
$2a^2-15ab$

17. $\frac{3x^7 \cdot 4x^9}{2x^3} = \frac{12x^{16}}{2x^3}$

$\boxed{6x^{13}}$

18. $\frac{(x^4)^5 \cdot 12x^5}{4x^7}$

$\frac{x^{20} \cdot 12x^5}{4x^7} = \frac{12x^{25}}{4x^7} = \boxed{3x^{18}}$



★17+18 are different than the remaining questions because you can multiply unlike terms, but you can't add or subtract unlike terms.

Notes 12.3 - Multiplying Polynomials by Polynomials

Multiply $(x + 4)(x - 3)$. ① Double distribute. or ② Geometric model (multiply = find area)

$x(x-3)+4(x-3)$

$x^2-3x+4x-12$

x^2+x-12

	x	-3
x	x^2	$-3x$
4	$4x$	-12

x^2+x-12

Try these:

1. $(p + 3)(p + 7)$

$p^2+7p+3p+21$

$p^2+10p+21$

2. $(5h - 2)(3h + 8)$

	$5h$	-2
$3h$	$15h^2$	$-6h$
8	$40h$	-16

$15h^2+34h-16$

3. $(c - 6)(c - 2)$

$c^2-2c-6c+12$

$c^2-8c+12$

4. $(w - 3)(w + 3)$

	w	-3
w	w^2	$-3w$
3	$3w$	-9

w^2-9

5. $(x^2 + 2x + 1)(3x + 5)$

$3x^3+5x^2+6x^2+10x+3x+5$

$3x^3+11x^2+13x+5$

6. $(5a - 8)(5a + 8)$

	$5a$	-8
$5a$	$25a^2$	$-40a$
8	$40a$	-64

$25a^2-64$

7. $(7x - 3)^2$

$(7x-3)(7x-3)$

$49x^2-21x-21x+9$

$49x^2-42x+9$

8. $(2x^2 + 5x - 1)^2$

$(2x^2+5x-1)(2x^2+5x-1)$

	$2x^2$	$5x$	-1
$2x^2$	$4x^4$	$10x^3$	$-2x^2$
$5x$	$10x^3$	$25x^2$	$-5x$
-1	$-2x^2$	$-5x$	1

$4x^4+20x^3+21x^2-10x+1$

9. $(x + 3)^2$

$(x+3)(x+3)$

$x^2+3x+3x+9$

x^2+6x+9

Notes 12.4 - Factors and GCF

GCF stands for Greatest Common Factor

Find the GCF of the following. Then divide each number by the GCF.

1.) 18, 24

GCF: 6

Divided by the GCF: 3, 4

18: 1|18, 2|9, 3|6 — 24: 1|24, 2|12, 3|8, 4|6

2.) 13, 26

GCF: 13

Divided by the GCF: 1, 2

Finding GCF's of monomials:

A. Find the GCF of the coefficients (the number in front of the variable).

B. For each variable, look for the lowest exponent in common with each base given in each monomial.

3.) x^4, x^7

GCF: x^4

Divided by the GCF: $\frac{x^4}{x^4}$, $\frac{x^7}{x^4}$

4.) $8x^3, 4x^2, 2xy$

GCF: $2x$

Divided by the GCF: $\frac{4x^2}{8x^3}$, $\frac{2x}{4x^2}$, $\frac{y}{2xy}$

Practice Problems

1.) 2,4

GCF: 2

Divided by the GCF: 1, 2

2.) 14,21

GCF: 7

Divided by the GCF: 2, 3

3.) 12, 21, 9

GCF: 3

Divided by the GCF: 4, 7, 3

4.) 12, 16, 18

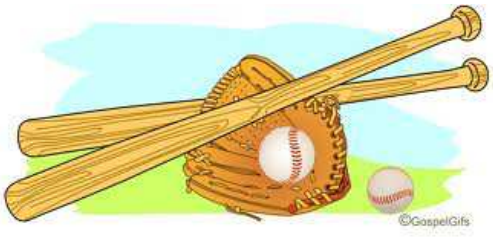
GCF: 2

Divided by the GCF: 6, 8, 9

5.) x^6, x^7, x^8

GCF: x^6

Divided by the GCF: 1, x, x^2



6.) $3xy^3, 9x^2y^2, 18x^3y$

GCF: $3xy$

Divided by the GCF: y^2, $3xy$, $6x^2$

7.) $4ab^2, 7abc$

GCF: ab

Divided by the GCF: 4c, 7c

Notes 12.5 - Factoring by GCF

1.) Distribute: $3x(x^3+8x^2+4)$

$3x^4+24x^3+12x$

2.) What is the GCF of each term in your answer?

$3x$ (What happens when you divide the answer to #1 by the GCF?)

$\frac{3x^4+24x^3+12x}{3x}$

x^3+8x^2+4 WOW!

Steps to Factoring by GCF:

1.) Find the GCF.

2.) Divide by the GCF.

3.) Write your answer as: GCF (quotient)

3.) $4x^4+24x^3+12x$

$4x(x^3+8x^2+3)$

(If you multiply through, you'll get the original problem! :))

4.) $3xy^3-9x^2y^2+18x^3y$

$3xy(y^2-3xy+6x^2)$

Practice Problems

1.) $5x+5$

$5(x+1)$

2.) $12x-24$

$12(x-2)$

3.) $16x^2-4x$

$4x(4x-1)$

4.) $8x^3-4x^2+2x$

$2x(4x^2-2x+1)$

(Quotient should have same number of terms as original problem)

5.) $25x+30$

$5(5x+6)$

6.) $17x^2y^2z-38xy$

$17xy(xyz-2)$

7.) $189x^3-108x^2+162x$

$27x(7x^2-4x+6)$

8.) $4x^2y+7xy$

$xy(4x+7)$

Notes 12.6 - Factoring Trinomials

Trinomial:

3 algebraic terms

Multiply the following binomials.

1.) $(x+3)(x+5)$

	x	3
x	x^2	3x
5	5x	15

$x^2+8x+15$

2.) $(x-4)(x-8)$

	x	-4
x	x^2	-4x
-8	-8x	32

$x^2-12x+32$

3.) $(x-2)(x+12)$

	x	-2
x	x^2	-2x
12	12x	-24

$x^2+10x-24$

What do you notice? If you add the constants in the original expression you get the coefficient of the second term in the trinomial and if you multiply them, you get the third term in the trinomial

Fill in the signs for the following factored expressions:

$x^2+9x+14$	$(x+7)(x+2)$ $(+7)(+2)=+14$ $+7+2=+9$
$x^2-12x+11$	$(x-11)(x-1)$ $(-11)(-1)=+11$ $-11-1=-12$
$x^2+2x-24$	$(x+6)(x-4)$ $(+6)(-4)=-24$ $+6-4=+2$
$x^2-8x-33$	$(x-11)(x+3)$ $(-11)(+3)=-33$ $-11+3=-8$

Factor the following trinomials.

1.) a^2+6a+8

a^2 8 — 8: 1|8, 2|4

$(a+2)(a+4)$

$2a+4a=6a$

$(a+2)(a+4)$

2.) $12x^2+5x-2$

Guess + check — 12: 1|12, 2|6, 3|4 — 2: 1|2

Trial #1: $(12x\ 1)(x\ 2)$

$12x+x=24x$

Doesn't give 5x. Try again!

Trial #2: (switch green 1+2)

$(12x\ 2)(x\ 1)$

$2x+12x$ Nope!

Trial #3: $(6x\ 1)(2x\ 2)$

$2x+6x$ No!

Trial #4: $(6x\ 2)(2x\ 1)$

$4x+6x$ No

Trial #5: $(4x-1)(3x+2)$

$-3x+8x$ $8x-3x=5x$!

This works because $4x\cdot 3x=12x^2$, $8x-3x=5x$, and $(-1)(2)=-2$!

$(4x-1)(3x+2)$

Practice Problems

3.) $b^2 + 10b + 21$

$(b+7)(b+3)$

$7b + 3b$

21: 1|21, 3|7

4.) $2x^2 + 7x + 3$

$(2x+1)(x+3)$

$+x + 6x = 7x$

2: 1|2 3: 1|3

5.) $x^2 + 4x - 45$

$(x+9)(x-5)$

$+9x - 5x$

45: 1|45, 3|15, 5|9

6.) $3x^2 - 11x + 10$

$(3x-5)(x-2)$

$-5x - 6x = -11x$

3: 1|3 10: 1|10, 2|5

7.) $x^2 + x - 12$

$(x+4)(x-3)$

$+4x - 3x = +1x$ (or x)

12: 1|12, 2|6, 3|4

8.) $x^2 - x - 90$

$(x-10)(x+9)$

$-10x + 9x$

90: 1|90, 2|45, 3|30, 5|18, 9|10

9.) $3x^2 - 10x - 8$

$(3x+2)(x-4)$

$+2x - 12x = -10x$

3: 3|1 8: 1|8, 2|4

Note:
$(3x+4)(x-2)$

$+4x - 6x$ This doesn't work b/c to get $-10x$ from $4x$ and $6x$, both $4x$ and $6x$ must be negat
This means that 4 and 2 must both be negative. But $(-4)(-2) = +8$
and we need -8.

10.) $5x^2 + 28x + 15$

5: 1|5 15: 1|15, 3|5

$(5x+3)(x+5)$