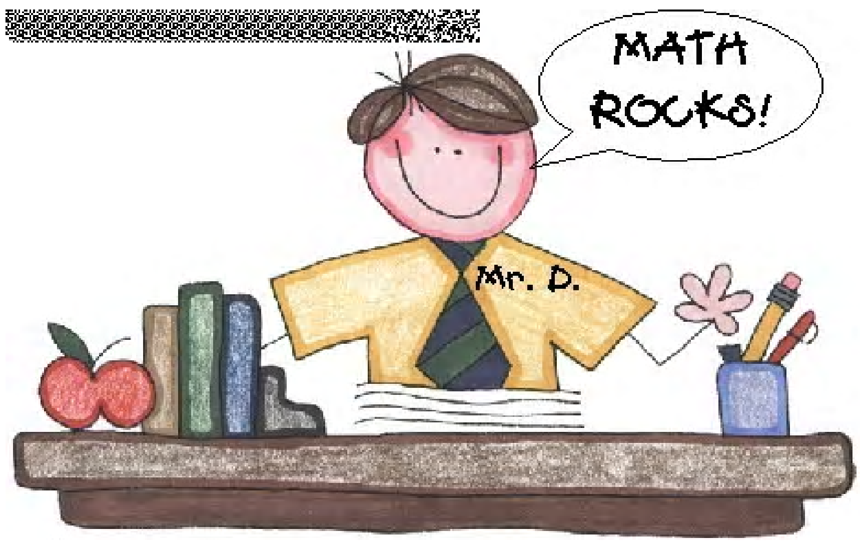
Unit 5 Notes

Systems of Equations



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Tentative Schedule

Day	*Date*	*Classwork*	*Assignment*
	Thurs. 12/11	Test #4	Watch Video #5.1 with Notes – Solving Systems of Equations Graphically
1	Fri. 12/12 Mon. 12/15	Introduction to Solving Systems by Substitution	P.S. #5.1
2	Tues. 12/16	Continue Solving Systems by Substitution	Watch Video #5.3 with Notes – Solving Systems of Equations by Substitution
3	Wed. 12/17 Thurs. 12/18	Introduction to Solving Systems by Elimination	Finish P.S. #5.3
4	Fri. 12/19	Continue Solving Systems by Elimination	Finish P.S. #5.4 Watch Video #5.5 – Solving Systems of Equations by Elimination
5	Mon. 1/5 Tues. 1/6	Activity	Finish P.S. #5.5
6	Wed. 1/7	**Quiz #5**	Video #5.6 – Applications of Systems of Equations
7	Thurs. 1/8 Fri. 1/9	Practice Applications of Systems of Equations	Finish P.S. #5.6
8	Mon. 1/12	Activity Catch-up Day	Catch-up
9	Tues. 1/13 Wed. 1/14	Special Cases of Systems of Equations	P.S. #5.7
10	Thurs. 1/15	Review for Test #5	Review for Test #5
11	Fri. 1/16 Tues. 1/20	Review for Test #5	Review for Test #5
12	Wed. 1/21	**Test #5**	

Notes 5.1 - Solving Systems Graphically

1.) Circle all ordered pairs (x,y) that are solutions to the equation $4x - y = 10$.

x y (3,2) | x y (2,3) | x y (-1,-14) | x y (0,-10) | x y (3,4)

4x - y = 10
4(3) - 2 = 10
12 - 2 = 10
10 = 10 ✓

4(2) - 3 = 10
8 - 3 = 10
5 ≠ 10

4(-1) - -14 = 10
-4 + 14 = 10
10 = 10 ✓

4(0) - -10 = 10
0 + 10 = 10
10 = 10 ✓

4(3) - 4 = 10
12 - 4 = 10
8 ≠ 10

2.) Find another solution to $4x - y = 10$.

4(7) - 18 = 10
28 - 18 = 10
10 = 10 ✓

(7,18)
(answers vary)

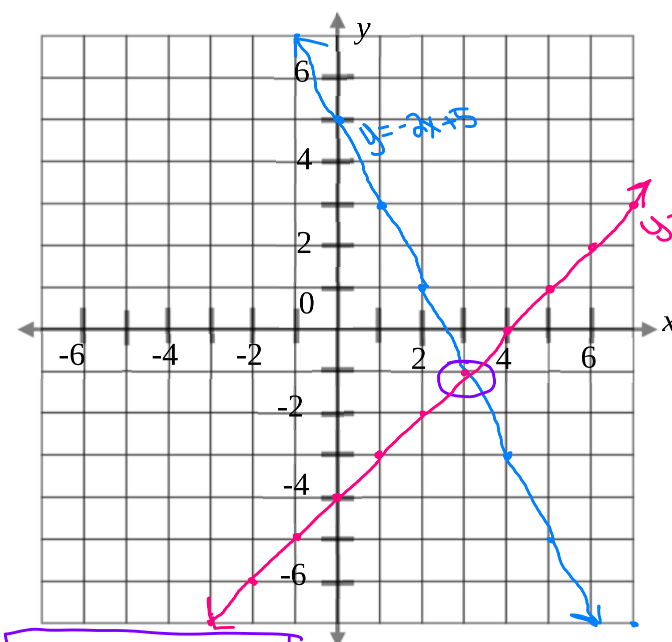
3.) How many solutions are there to $4x - y = 10$?

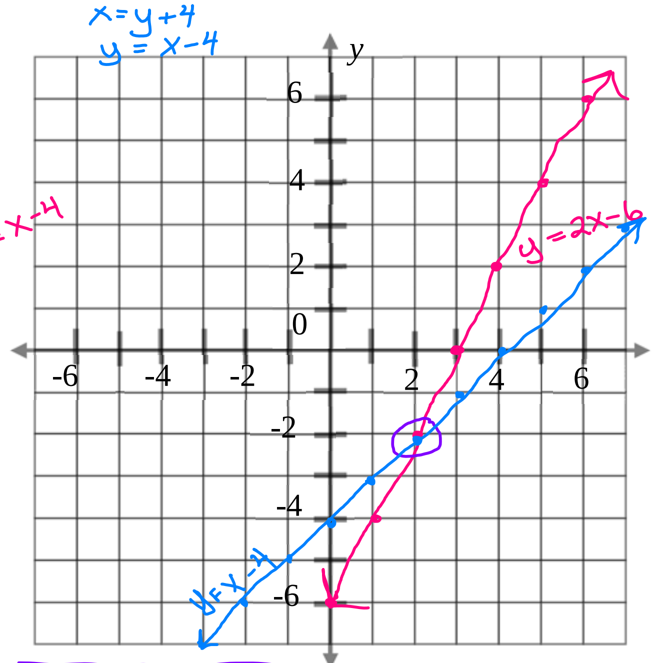
infinite (infinite points make a line.)

System of Equations: a set of equations that have one simultaneous solution.

4.) $y = x - 4$
$y = -2x + 5$

5.) $3y + 18 = 6x$ → $3y = 6x - 18$
$x - y = 4$ → $y = 2x - 6$
$x = y + 4$
$y = x - 4$





Solution: (3,-1)

Solution: (2,-2)

Check:

y = x - 4
-1 = 3 - 4
-1 = -1 ✓

y = -2x + 5
-1 = -2(3) + 5
-1 = -6 + 5
-1 = -1 ✓

Check: (in original equations)

3y + 18 = 6x
3(-2) + 18 = 6(2)
-6 + 18 = 12
12 = 12 ✓

x - y = 4
2 - -2 = 4
4 = 4 ✓

Notes 5.2 - Solving Systems Using Substitution Day 1

Given the following system of equations, solve for x and solve for y.

$3x - 2y = 4$
$x = 2$

$3x - 2y = 4$
$3(2) - 2y = 4$
$6 - 2y = 4$
$-6 \quad\; -6$
$-2y = -2$
$y = 1$

$\boxed{(2,1)}$

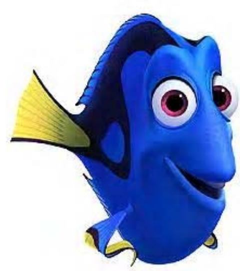
Solve the following system of equations.

$y = 3x$
$2x + 5y = 34$

$2x + 5y = 34$
$2x + 5(3x) = 34$
$2x + 15x = 34$
$17x = 34$
$x = 2$

$y = 3x$
$y = 3(2)$
$y = 6$

$\boxed{(2,6)}$



Solve the following system of equations.

$x = 2y + 2$
$4x + 3y = 41$

$4x + 3y = 41$
$4(2y + 2) + 3y = 41$
$8y + 8 + 3y = 41$
$11y + 8 = 41$
$-8 \quad -8$
$11y = 33$
$y = 3$

$x = 2y + 2$
$x = 2(3) + 2$
$x = 6 + 2$
$x = 8$

$\boxed{(8,3)}$

Notes 5.3 - Solving Systems Using Substitution Day 2

Solve the following system of equations:

$-4x + y = 6$
$-5x - y = 21$

① $-4x + y = 6$
$+4x \quad +4x$
$y = 6 + 4x$

② $-5x - y = 21$
$-5x - (6+4x) = 21$

③ $-5x - 6 - 4x = 21$
$-9x - 6 = 21$
$+6 \quad +6$
$-9x = 27$
$x = -3$

④ $-4x + y = 6$
$-4(-3) + y = 6$
$12 + y = 6 \rightarrow y = -6$

⑤ $(-3, -6)$

List of Steps to Solve a System of Equations by Substitution:

$3x - 2y = 11$
$x + 2y = 9$

Steps	*Solution*
1.) Isolate a variable in one equation. Look for the easiest variable to isolate!	$x + 2y = 9$ $-2y \quad -2y$ $x = 9 - 2y$
2.) ***Substitute*** that into the other equation.	$3x - 2y = 11$ $3(9-2y) - 2y = 11$
3.) Now that you only have one variable in the equation, solve it.	$27 - 6y - 2y = 11$ $27 - 8y = 11$ $-27 \quad -27$ $-8y = -16 \rightarrow y = 2$
4.) Plug the answer into any equation to find the other variable.	$3x - 2y = 11$ $3x - 2(2) = 11$ $3x - 4 = 11$ $3x = 15$ $x = 5$
5.) Write your answer as a coordinate.	$(5, 2)$
6.) Check the solution in *both* equations.	$3x - 2y = 11$ $3(5) - 2(2) = 11$ $15 - 4 = 11$ $11 = 11$ ✓ $x + 2y = 9$ $5 + 2(2) = 9$ $5 + 4 = 9$ $9 = 9$ ✓

Notes 5.4 - Solving Systems Using Elimination Day 1

- Just like substitution, we want to end up with an equation with only one variable. Using this method, we eliminate a variable by adding the equations.
- Make sure the signs are opposites.
- Make sure your variables line up before you add!

$x + 2y = 8$
$x - 2y = 4$

You are going to work with your partners to determine a possible solution to solving the system of equations above. It does not matter if your answer is right or wrong. What matters it that you **persevere and you take risks**.

$x + \cancel{2y} = 8$ eliminated
$+\; x - \cancel{2y} = 4$
$2x = 12$
$\frac{2x}{2} = \frac{12}{2}$
$x = 6$

$x + 2y = 8$
$6 + 2y = 8$
$-6 \qquad -6$
$2y = 2$
$\frac{2y}{2} = \frac{2}{2}$
$y = 1$

$\boxed{(6,1)}$

1.) $-(x + y = 18)$
$x + 2y = 25$

$-\cancel{x} - y = -18$
$\cancel{x} + 2y = 25$
$y = 7$

$x + y = 18$
$x + 7 = 18$
$-7 \quad -7$
$x = 11$

$\boxed{(11,7)}$

2.) $3x - \cancel{5y} = 3$
$4x + \cancel{5y} = 4$

$7x = 7$
$\frac{7x}{7} = \frac{7}{7}$
$x = 1$

$4x + 5y = 4$
$4(1) + 5y = 4$
$4 + 5y = 4$
$-4 \qquad -4$
$5y = 0$
$y = 0$

$\boxed{(1,0)}$

Notes 5.5 - Solving Systems by Elimination Day 2

$4x+3y=-1$
$5x+4y=1$

You are going to work with your partners to determine a possible solution to solving the system of equations above. It does not matter if your answer is right or wrong. What matters it that you **persevere and you take risks**.

$-4(4x+3y=-1) \rightarrow -16x-12y=4$
$3(5x+4y=1) \rightarrow 15x+12y=3$

$-x=7$
$\frac{-x}{-1}=\frac{7}{-1}$
$x=-7$

$4x+3y=-1$
$4(-7)+3y=-1$
$-28+3y=-1$
$+28 \quad +28$
$3y=27$
$y=9$

$\boxed{(-7,9)}$

1.) $9(x+y=14) \rightarrow 9x+\cancel{9y}=126$
$9x-9y=36 \rightarrow 9x-\cancel{9y}=36$

$18x=162$
$x=9$

$x+y=14$
$9+y=14$
$-9 \quad -9$
$y=5$
$\boxed{(9,5)}$

2.) $3y=-2x+5$
$+2x \quad +2x$
$2(5x+4y=16) \rightarrow 10x+8y=32$
$-5(2x+3y=5) \rightarrow -10x-15y=-25$

$-7y=7$
$y=-1$

$5x+4y=16$
$5x+4(-1)=16$
$5x-4=16$
$5x=20$
$x=4$

$\boxed{(4,-1)}$

Notes 5.6 - Applications of Systems of Equations

Do not forget to write let statements!

1.) Alexa purchased 12 pens and 14 notebooks for $20. Hannah bought 7 pens and 4 notebooks for $7.50. Find the price of one pen and the price of one notebook, algebraically.

Let cost of pen = p
Let cost of notebook = n

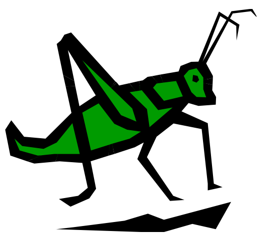
$2(12p+14n=20) \rightarrow 24p+\cancel{28n}=40$
$-7(7p+4n=7.5) \rightarrow -49p-\cancel{28n}=-52.5$
$-25p=-12.5$
$p=0.5$

$12p+14n=20$
$12(0.5)+14n=20$
$6+14n=20$
$14n=14$
$n=1$

One pen costs $0.50 and one notebook costs $1.00.

2.) Tyler has a collection of grasshoppers and crickets. He has 561 insects in all. The number of grasshoppers is twice the number of crickets. Find the number of *each* type of insect that he has.

Let # of grasshoppers = g
Let # of crickets = c



$\left.\begin{array}{l} g+c=561 \\ g=2c \end{array}\right\}$ substitution

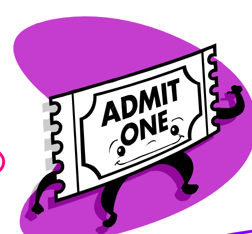
$2c+c=561$
$3c=561$
$c=187$
$g=2c=2(187)=374$

187 crickets and 374 grasshoppers

3.) A total of 600 tickets were sold for a concert. If the tickets sold in advance cost $25 each and the tickets sold at the door cost $32 each, and $16,309 worth of tickets was sold, how many of each type of ticket was sold?

Let # of advance tix = a
Let # of door tix = d

$-25(a+d=600) \rightarrow -25a-25d=-15000$
$25a+32d=16309 \rightarrow 25a+32d=16309$
$7d=1309$
$d=187$
$a+d=600$
$a+187=600$
$a=413$



413 tickets sold in advance and 187 tickets sold at the door.

Notes 5.7 - Special Cases of Systems of Equations

Warm-up: Please solve the following equations.

You have learned to find the unique solution to a system of linear equations, when it exists. However, not every system of linear equations has a unique solution.

1.) $3(x+4) = 2x + 17 + x - 5$

$3x + 12 = 3x + 12$

∞ solutions

x can be any $\mathbb{R}$

2.) $2(x+5) + 3x = 5x + 14$

$2x + 10 + 3x = 5x + 14$

$5x + 10 = 5x + 14$
$-5x \qquad -5x$

$10 \neq 14$

no solution

null set

3.) With your partners, please solve the following system of equations using *substitution*.

$2x + y = 1 \rightarrow$
$4x + 2y = 4$

$2x + y = 1$
$-2x \qquad -2x$

$y = 1 - 2x$

$4x + 2y = 4$

$4x + 2(1 - 2x) = 4$

$4x + 2 - 4x = 4$

$2 \neq 4$

$\varnothing$

4.) With your partners, please solve the following system of equations using *elimination.*

$-2(2x + y = 1) \rightarrow \quad -\cancel{4x} - \cancel{2y} = -2$
$4x + 2y = 4 \rightarrow \quad \cancel{4x} + \cancel{2y} = 4$

$0 = 2$

$\varnothing$

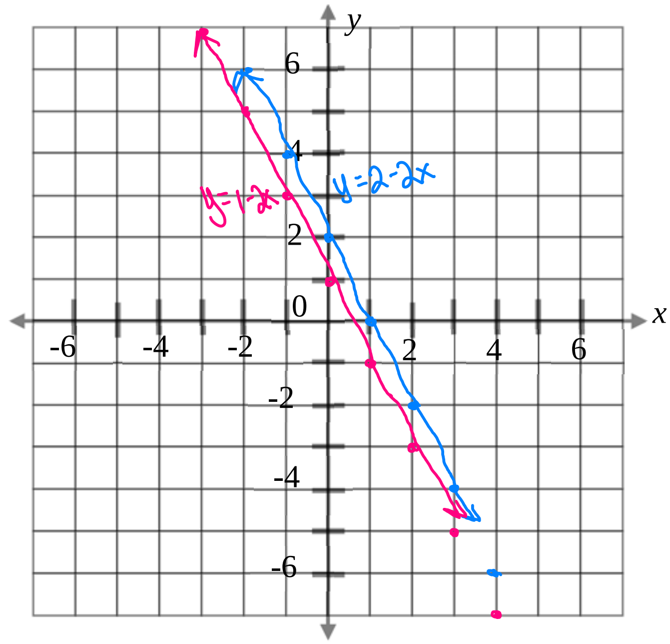
5.) With your partners, please solve the following system of equations by *graphing*.

$2x + y = 1$

$4x + 2y = 4$

$2x+y=1$
$-2x \quad -2x$
$y=1-2x$

$4x+2y=4$
$-4x \quad -4x$
$\frac{2y}{2}=\frac{4}{2}-\frac{4x}{2}$
$y=2-2x$



6.) What happened when you tried to solve the equation with all three methods?!

There were no solutions!

7.) With your partners write a **thorough explanation** why this happened algebraically.

The variables are the same, (when the equations are simplified) but the constants are different, so it's impossible to have the same variables equal to different constants.

8.) With your partners write a **thorough explanation** why this happened graphically.

They have the same slope (same variables), but different y-intercepts (different constants), so the lines are parallel and therefore never intersect.

9.) With your partners, please solve the following system of equations using *substitution*.

$x + 2y = 2$

$2x + 4y = 4$

$x+2y=2$
$-2y \quad -2y$
$x=2-2y$

$2x+4y=4$
$2(2-2y)+4y=4$
$4-4y+4y=4$
$4=4$

∞ solutions

10.) With your partners, please solve the following system of equations by ***graphing***.

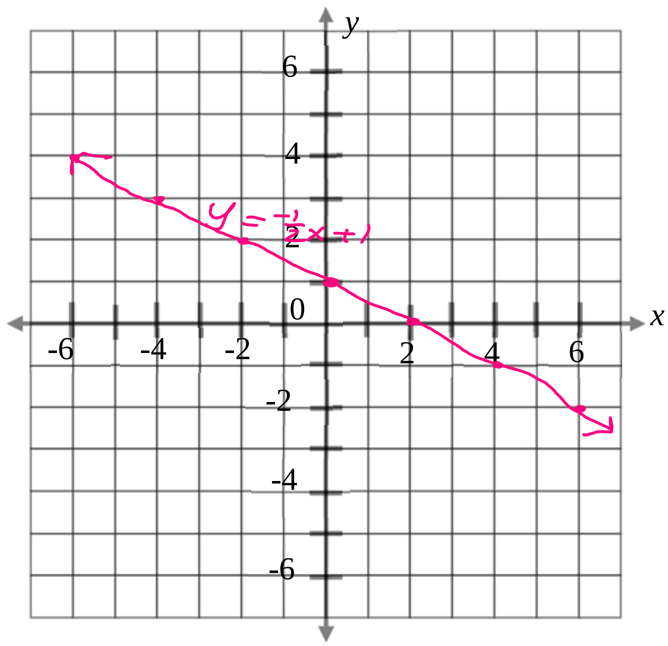
$x+2y=2$

$2x+4y=4$

$x+2y=2$
$-x \quad -x$
$\frac{2y}{2}=\frac{-x}{2}+\frac{2}{2}$
$y=-\frac{1}{2}x+1$

$2x+4y=4$
$-2x \quad -2x$
$\frac{4y}{4}=\frac{-2x}{4}+\frac{4}{4}$
$y=-\frac{1}{2}x+1$

They're the same line - so they intersect ∞ times.



11.) With your partners, please solve the following system of equations using ***elimination.***

$-2(x+2y=2) \rightarrow -2x-4y=-4$

$2x+4y=4 \rightarrow 2x+4y=4$

$0=0$

∞ solutions

12.) What happened when you tried to solve the equation with all three methods?!

There were ∞ solutions

13.) With your partners write a **thorough explanation** why this happened with algebraically.

When the equations are simplified, they are exactly the same, so any set of ordered pairs that satisfy one equation will satisfy the other equation.

14.) With your partners write a **thorough explanation** why this happened graphically.

When the equations are simplified, they are exactly the same, so they'll both have the same graph. Each line will intersect with the other line everywhere.

Summary:

- There is no solution when the variables are the same, but the constants are different (algebraically) or the lines are parallel (graphically).

Example:

$2x+3y=1$
$2x+3y=9$

- There is one unique solution when the variables are different (algebraically) or the lines intersect (graphically).

Example:

$2x+3y=1$
$5x+7y=9$

- There are infinite solutions when the variables and the constants are the same or the lines are the same (graphically).

Example:

$2x+3y=1$

$4x+6y=2$